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Analysis of mutual funds performance across the five European economies with the largest amount of financial assets

Análisis del rendimiento de fondos de inversión en las cinco economías europeas con un mayor número de activos financieros

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Abstract:

This project studies mutual funds performance to explain the possible selection of mutual funds as an investment instrument with its own characteristics. The study is performed at European level and analyzes the situation in the five community countries with the largest amount of capital invested in financial assets: United Kingdom, France, Germany, Italy and Spain. The performance analysis of the domestic equity mutual funds in these countries comprises the ten-year period between 2008 and 2017 to assess the evolution of these funds in the financial crisis and the following recovery. We also carry out an individual portfolio holding analysis of the best performing mutual funds in each country and find that the best performers develop different portfolio holding strategies.

Resumen:

Este proyecto estudia el rendimiento de los fondos de inversión para explicar su posible uso como un instrumento de inversión con sus características propias. El estudio se realiza a nivel europeo y analiza la situación en los cinco países comunitarios con un mayor capital invertido en activos financieros: Reino Unido, Francia, Alemania, Italia y España. El análisis de estos fondos domésticos de renta variable comprende diez años entre 2008 y 2017, observando la evolución durante la crisis financiera y la siguiente recuperación económica. A su vez, se realiza un análisis individual la composición de los fondos de inversión con un mejor rendimiento en cada uno de los países, encontrando cómo estos fondos desarrollan diferentes estrategias en su composición.

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1. INTRODUCTION

The current economy is defined by the interrelation between countries and regions. The events and policies taken in one country can have major effects in the rest of the world. One of the main factors in this economic globalization has been worldwide financial markets. Financial markets enable the flow of funds from individual savers to corporations and, therefore, its good functioning is essential for the whole economy. Individual savers have many ways to take part of the financial market and get extra profit from their capital. Among those instruments, mutual funds are one of the most important available options for investors.

Many individual investors believe that mutual funds are investment vehicles for savers with a large amount of capital; however, even if most mutual funds have minimum capital requirements, there are funds available for every investor. In this project, we explain the importance of mutual funds and its functioning, showing that mutual funds can be used by any individual investors. Specifically, this project analyzes the fund performance of domestic equity mutual funds and the diversification strategy carried out in the fund investment in five different countries: France and Germany, representing continental European countries, Spain and Italy, representing Mediterranean countries, and the United Kingdom, the most representative Anglo-Saxon country. The election and analysis of five different countries will help to understand and compare Spanish mutual funds with the other four countries selected.

The aforementioned analysis is presented in the first section of the dissertation. The period considered for this analysis includes the last 10 years, starting in 2008 with the beginning of the financial crisis, and finishing with the economic recovery that Europe experiences. The analysis of the macroeconomic situation is included in the second section of the project. The next part of the project is focused on the performance of mutual funds and begins with the explanation of the theoretical models used, mainly the Capital Asset Pricing Model and the optimization problem proposed. By using the Capital Asset Pricing Model, we examine the performance of mutual funds in every country. Those funds with better performance are considered in the optimization problem to test if they were profit maximizers or risk minimizers. The project concludes representing these results, comparing them across countries and defining if there are important performance differences between countries and identifying the reasons of these results. The last section includes the conclusions.

2. THE FINANCIAL MARKET AND MUTUAL FUNDS.

Mutual funds are one of the most relevant financial instruments globally with a large influence in financial markets. We should first define what we consider as financial markets to next define mutual funds. A financial market could be defined as the platform through different economic agents – individuals and legal entities – flow their money into a set of financial assets that can range among government and corporate bonds, corporations' shares, or different currencies, among others. The financial market amplifies the interconnection between different markets and provides individuals and companies access to financing and investments that would not be available otherwise. There are different ways in which individuals can participate in financial markets and look for profitability in the market, besides the most conservative solutions of keeping the money in bank deposits (with current negative real returns in several European countries). The most popular financial instruments are direct investments, as pension funds, insurances, and mutual funds; the latter are the object of this study.

Investment funds are a pool of capital provided by different investors or institutions to purchase financial assets jointly. The first investment fund was created less than a century ago, in 1924. An investment fund provides investors with the access to more opportunities in the market that would not be available in the case that they were acting as single investors, allowing them to reduce market fees by achieving economies of scale and, most importantly, they are coordinated by a professional expert who is responsible for the management of the fund. However, investment fund is a very broad term with a lot of different funds grouped under this term that should be distinguished. Some of these funds are money market funds (investing in treasury securities and bonds), hedge funds (more aggressively run, usually applying derivatives and leverage strategies), exchange-traded funds or ETFs (replicating the market or a sector in the market) and mutual funds.

According to the latest data, released by INVERCO for the year 2017 (INVERCO 2018), it is estimated that the total amount of equity in investment institutions reached 41 billion Euros (50 billion USD). If we look at the total amount of assets invested in the five countries analyzed, Spain is the one in which its population invests a lower amount in financial assets per capita (approximately 60,000 €), while Germany, France and Italy are in a similar position (approximately 80,000 €), and the United Kingdom is

the country in which its population owns the largest amount of financial assets per capita (approximately 120,000 €).

Mutual funds are a particular type of investment fund in which the owners of the fund invest in shares, fixed securities and/or the money market collectively. These funds are run by a professional manager who tries to obtain returns above the market index, used as benchmark, producing gains for the investor. Their portfolios are structured and kept in accordance with the fund prospectus and the portfolio information that is publicly available. The investment capital of a mutual fund is split in units or shares owned by the investors of the fund, and their value is updated daily and called Net Asset Value (NAV). The NAV represents the value at which a fund share would be sold or purchased. Investors can enter and exit of the fund at any moment by acquiring or redeeming these shares; therefore, the capital of the fund will flow according to the investment decisions of investors. The value of these shares will be determined by the performance and evolution of the assets invested by the fund. The main types of mutual funds are: fixed income funds (investing in government or corporate bonds), indexed funds (trying to replicate the composition of a market index), equity funds (investing mainly in equity), and balanced or mixed funds (composed by both fixed income and equity securities).

One of the main advantages of taking part in a mutual fund is that individual investors access a diversification level that would be hardly achievable in case that they were acting as single investors. Other advantages are the professional experience of the fund manager, who has better access to information that would be hardly obtained in case that investors were acting alone. In addition, funds have different styles, according to the needs and preferences of investors, being able to select between value and growth asset investments, or domestic versus international investments, among others, without having to carry out their own individual research. However, mutual funds also present disadvantages and risks. One of them is the return risk as the future performance and returns cannot be guaranteed by past performance, as well as market conditions are subject to variations at any given moment. Moreover, the investment in a mutual fund is not backed by central banks, as it is in the case of deposits if banks declare insolvency. Additionally, the costs borne by investors must be evaluated when deciding about getting into these funds. Specifically, the most common fees are management fees, which should be paid periodically for the professional management, and front-end or

back-end fees, which should be paid before the investment or when taking it out, respectively, as well as taxes over the capital gains.

Consequently, mutual funds are a relevant investment option that investors should consider due to the current context, giving that investors can rely their investment in a professional fund manager. Nevertheless, investors should know their risks, the fact that profits are not guaranteed, many of these funds will also have a performance similar to that of the index market of reference (benchmark) and with very low values of alpha, as managers do not desire to obtain lower return than their main benchmark.

After describing the general characteristics of mutual funds, we proceed to explain the mutual fund markets studied in this work.

2.1. Mutual funds in Spain.

Among the countries included in the study, Spain is the one in which the lowest amount of savings is placed into these investment instruments per capita. There are two main reasons behind this result: the lower income per capita in Spain compared, mainly, with Germany and the United Kingdom, but also and most important, the higher investment of Spanish individuals in real estate. In fact, Spain is the second country in the European Union with a higher real estate ownership percentage.

The investment in financial assets of the Spanish population has changed in the last ten years and should be analyzed to understand the evolution and importance of mutual funds in the country. During the last ten-year period, deposits have been the preferred financial instruments of Spanish savers, but the position in liquid and safe assets – deposits, cash and other instruments - has been decreasing along the period from 49.8% of the total financial assets in 2008 to 42.2% in 2017 due to the low interest rates. The reduction in liquid assets has flown mostly to institutional investors, as their share has increased 6 percentage points in just 10 years, from 8.9% in 2008 to 14.8% in 2017.

The total investment in Spanish domestic mutual funds in December 2017, according to Inverco, reaches a total of 262,847 million Euros, which is the highest value recorded, surpassing the previous record of May 2007, with a total of 261,073 million Euros, and recovering from the lowest point of 122,322 million Euros in December 2012. Therefore, the amount invested in domestic funds has doubled in Spain in just five years, partly because of the revaluation of the Spanish stock market and, more importantly, due to higher subscription values. In fact, during this five-year period, 84%

of this increase has been due to new net subscriptions and 16% is due to the evolution of the stock market in this period, which shows how net subscriptions are more important in the evolution of the fund total capital than the performance of the fund itself.

Attending at the composition of the funds, it has changed significantly in the last ten years, from a majority of funds invested in short-term debt and the money market (36.6%), and passive management (23.5%) in 2007, to a more diversified portfolio, dominated by mixed assets (24.9%), market securities (13.7%), and global assets (20.1%).

Finally, we should also consider the taxation of the profit obtained from the investment. The profits obtained are not taxed until the investment is effectively recovered from the fund by investors, and not when the investor is transferring her money from one fund to another. In case the profit is effectively cashed out, investors are subject to tax under the savings account in the IRPF (Impuesto sobre la Renta de las Personas Físicas), which is the Spanish individual tax. The due amount depends on the amount obtained; specifically, 19% if the profit is lower than 6,000€, 21% if the profit range between 6,000 and 50,000€, and 23% for those revenues over 50,000€. There are some exceptions applicable for the regimes present in Navarre and the Basque Country decided by their regional governments[1].

2.2. Mutual funds in Italy.

Italian investors are characterized for their safe and conservative approach regarding financial decisions. Historically, Italy ranks as one of the countries in the European Union with higher savings rate and lower private debt. On the other hand, the effects of the financial and the government bonds crises have had important effects on Italian savers. In fact, the savings rate has decreased from a pre-crisis 14% level (over the Eurozone average) to a level over 10%, which is around 2 points lower than the average of the Eurozone. The positive value for Italian households is the low indebtedness percentage that stands at 60%, 20 points below the next country in this study: Germany.

This attitude caused the preference towards safe investments, such as deposits or low-risk government fixed securities. As a result, mutual funds were introduced in Italy later than in other countries, such as France or the United Kingdom. These financial assets became popular in the 1990s decade, and now are fully integrated as an option for

[1]Ley del Impuesto sobre la Renta de las Personas Físicas. Arts.66, 76.

Italian investors. In the last 10 years, Italian investors have decreased their position in risky financial investments. In 2008, 36.1% of the financial investment of Italian savers was devoted to investment funds and direct investment (shares), while this rate reached 33.5% in 2017, and the investment in pension funds has increased significantly in 8 points.

The investment in mutual funds in Italy is small compared to direct investment in shares, which almost doubles the capital invested in mutual funds. According to data provided by Inverco for 2017, 480,000 million Euros were invested in mutual funds, above the amount invested in Spain. The trend has been positive since 2011, but at a lower rate than in Spanish mutual funds, mainly for open-end funds[2]. Comparing this value to the pre-crisis situation, there has been a major decrease in the mutual fund investment, which has not been recovered yet, as one third of the assets under management were retired from these accounts in 2008.[3]

Attending to the composition of open-end funds and investment institutions, Italian investors show a tendency towards safe investments, and 66.3% of total assets are government and corporate bonds, while shares only represent 15.1% of the total investment. However, there has been a major change from the levels of 2008, when 74.4% of the total fund assets were fixed-income securities, and the fund investment in shares has increased 3 points, and indirect mixed investments have grown around 10 points[4].

With regard to the tax legislation in Italy, profits obtained from mutual funds are considered by the Italian public administration as financial income that should be subject to taxation. In 2018, the tax rate for the profit obtained from mutual funds, paid when the capital is recovered from the fund, amounts to 26% of the total profit, after the 2015's fiscal reform that pushed the rate up from 20%, though the rate for fixed-income instruments stands at 12.5%.[5] This tax does not have a progressive component as the Spanish rate does, but is proportional to the gains obtained, and it is one of the highest among the countries studied.

[2]Investment Company Institute, "2017 ICI Fact Book", P. 234
[3] Deloitte, "Asset management in Italy: a snapshot in an evolutive context", P. 9
[4] Mediobanca Richerce e Studi Spa, "Statistics on 1179 Italian Funds and SICAVs", P. 13
[5] https://www.segretibancari.com/formazione-e-guide/tassazione-rendite-finanziarie/

2.3. Mutual funds in France.

Mutual funds and alternative investment funds have an important presence in France. Open-end funds in France double the number of these funds in Germany and quadruple the number of these funds in Spain. It is especially significant since France has one of the largest per capita financial assets in Europe, which stands at €80.000. This result is due to an above Eurozone average savings rate at 14% of the total income and family indebtedness below 90% of the disposable income, below the Eurozone average. On the other hand, the savings rate has had a downwards trend, while the indebtedness rate has been increasing in the last 10 years.

French individual savers have a larger willingness to invest their capital in risk instruments compared to other European countries. In fact, over 40% of financial assets in French households correspond to investment funds and shares, while only 32% correspond to bank deposits and cash. In 2008, risk instruments accounted for just 34.7% of total financial assets. The increasing share of investment funds and shares in the period has been joined to a small decrease in deposits and a larger one in pension funds.

In global terms, during the third quarter of 2016, there were 871,200 million Euros invested in French mutual funds (INVERCO 2018). In addition, if we add the capital invested in shares, the total investment in risky financial assets reaches 2 billion Euros. If we look at the overall data from 2008, the total capital invested in mutual funds has decreased around 130,000 million Euros. The negative result for mutual funds and investment institutions is not exclusive of France due to the financial crisis. Because of the financial crisis, mutual funds in France lost one quarter of their total assets in 2008. After the sharp decrease in 2008, the capital invested in mutual funds increased steadily throughout the decade.

The total investment of French funds is split and allocated in different instruments. The most important fund assets are bonds and fixed return securities, which represented 49% in 2015, followed by 20% allocated in equities, and the rest in the money market or others. Taking as a reference the year 2007, the share of fixed-income securities has increased from 40% in 2007, while the equity fund share has lowered by 5%. These values are taken before the crisis (when equities were in record highs), which explains this trend, even though in recent years the fund share of equities has been growing due to the lower profitability obtained in bonds.

In France, the profit that investors get from their investment in mutual funds is subject to taxation when the actual distribution of the profit is done. These profits are taxed as if the investment was carried out directly by the investor and a progressive tax rate is applied. The French administration considers these gains as capital income that is exempt to the limit of € 9,710, and there are four different rate scales going from 14% (€ 9,711 – 26,818), 30% (€ 26,819 – 71,898), 41% (€ 71,899 – 152,260) to 44% for an income over € 152,261[6].

2.4. Mutual funds in Germany.

The financial investment as percentage of GDP in Germany is lower than in any other country of the group analyzed. Financial assets owned by German savers reach 178% of GDP, 10 points below Spain and more than 40 below France. The total investment per capita in financial assets in Germany amounts to around €75,000, being only higher than the Spanish value. These results are significant since Germany is the analyzed country with a higher level of GDP per capita. The conservative behavior of German investors, which save about 17% of their disposable income and whose private debt has constantly decreased during the last 10 years, shows why German investors invest a lower amount of their capital in financial assets than in France or the United Kingdom.

The distribution of financial assets of German investors shows their conservative attitude, since only 20% of total financial assets are invested in equity and risky investments (9.3% in mutual funds and 10.6% in shares), while almost 40% of those assets are deposits and liquid assets. In addition, pension funds have an important share over the total, which stands at 37.8 %, since German retired population only gets 42.5 % of the salary earned before their retirement. In the last 10 years, both pension funds and risky investments have grown in relevance between German investors by 4 points for the former and 5 for the latter, while fixed-income and other financial instruments have decreased significantly.

Aggregate data shows that the total investment in German mutual funds for 2016 amounts to 517.8 thousand million Euros (INVERCO 2018), that is, the third country after France and the United Kingdom in total terms, due to the size of German economy. In the case of listed equities owned by German investors, the value is at 590 thousand million Euros. Combining both, the result would still be 1 billion Euros lower

[6] Deloitte International Tax Source, "France Taxation and Investment 2017", P. 29

than the total investment in pension funds. If we analyze the evolution since 2009, the total investment in shares and mutual funds increased by 500 million Euros in the last eight years. The increase has been partly due to the higher share of these risky assets, but mostly due to the increase in over one billion Euros invested in Germany.

If we analyze the composition of German mutual funds, according to the Bundesbank, 51% of the fund assets are debt securities or fixed-income financial investments, 25% are shares, and 24% are other investment fund shares. It is also relevant to point out that most of German mutual funds are mixed security based (46%), followed by fixed securities funds (20%), while equity funds suppose 15% of the funds in Germany. [7]

Profits obtained by investors through mutual funds are subject to tax under the German legislation. The gains or losses from these investments are taxed once they are cashed, as in the rest of the European countries. This regulation is recorded in the German Investment Tax Act that was passed in 2016 and is effective since the beginning of 2018. Under this new legislation, investment funds are subject to corporate tax of 15% over dividends, capital gains and real estate gains. On the other hand, there are multiple exceptions to which mutual funds can apply.[8] From the investor point of view, the tax rate in Germany to which investment income and capital gains are subject to amounts to 25%.

2.5. Mutual funds in the United Kingdom.

The financial industry in the United Kingdom has a higher development degree than in any of the previously studied countries. The total capital invested in the UK financial market reaches 7,762.4 thousand million Euros, more than 2 billion higher than in Germany (INVERCO 2018). In terms of GDP, it represents 356% of the annual British GDP. If the data is analyzed at a single investor scope, financial assets per capita in the UK almost reach €120,000, a value significantly higher than in the continental European countries.

The composition of these financial assets is characterized by the dominance of pension funds. Capital invested in pension funds supposes 60% of total financial assets in the United Kingdom due to the importance of private savings for retirement. As a result, mutual funds and quoted shares represent 14.8% of total assets, which is a lower

[7]https://www.bundesbank.de/Redaktion/EN/Downloads/Statistics/Banks_Financial_Institutions/Investment_companies/factsheet_domestic_investment_funds.pdf?__blob=publicationFile
[8] Deloitte, "The reform of the German Investment Tax Act", P. 4

percentage than in any of the previously studied countries. In the last ten years, the dominance of pension funds has increased in 10 points from a level of 50% in 2008. Mutual funds and shares have also grown from the level of 5.6%. On the other hand, deposits and cash have decreased significantly. Therefore, the tendency towards riskier financial instruments has increased in the last decade.

The capital invested in mutual funds, as a share of total financial assets in the United Kingdom, presents one of the lowest values in Europe. However, analyzing the aggregate data, there is only more capital in these instruments in France. Specifically, there are 682.1 thousand million Euros invested in British mutual funds, while the total quoted shares in hands of British investors reach 467.2 thousand million Euros (INVERCO 2018). The capital invested in these instruments has almost increased 5 times the value of 2008. This result is due to the relative growth of mutual funds as an investment tool for British investors, but also due to the growth of the overall financial industry by over 4.3 billion Euros in ten years.

Analyzing the funds under management by asset type, British mutual funds are mainly focused on equities, representing 54.2% of the composition. Out of that value, 34.1% is represented by non-UK equities and the other 20.1% is invested in UK equities. Fixed-income assets represent just 17.6 % of the total, and mixed assets are 16.6 % of the total. The remaining assets are real estate or money market, which have a lower relative representativeness.[9] In the last ten years, overseas equities have increased its value, while the importance of UK equities decreased mainly in 2016, due to the Brexit uncertainty, and fixed income and mixed assets have been losing importance steadily.

The profit obtained from mutual funds is subject to taxation for investors. However, there is an exception for the first £ 11,300 and profits over this allowance will be the ones taxed. The British legislation differentiates between people with a higher or additional rate Income tax (income higher than £45,000 annually), which are subject to 20% tax on gains from these assets, from people that pay the basic rate Income tax (income lower than £45,000 annually), which are subject to 10% rate if they fall into the Income tax band, or 20% if they exceed it[10]. As a result, the United Kingdom has the lowest tax levels for mutual funds in the countries analyzed.

[9] The Investment Association, "Asset Management in the UK 2016-17", P. 63.
[10] Govern of the United Kingdom, "Capital Gains Tax."

3. MACROECONOMIC CONTEXT 2008-2017

In this section, we study the economic context in which the mutual funds studied have invested over the last ten years. The period 2008-2017 considered for this analysis has been full of economic and political events that have affected the performance of world stock indexes and, therefore, mutual funds across the globe. This section is divided into three parts. First, we analyze global events, followed by regional events in the scope of the European Union, specially the Eurozone, to compare those events that happened in continental Europe and the United Kingdom, finishing with those events particular of each country.

The beginning of the period considered for analysis was marked by the financial crisis that started in 2006-2007, mainly in the United States, and that started spreading around the globe. In this early stage, some European banks were starting to struggle. The French bank BNP Paribas had to freeze 1.6 billion Euros in August 2007 from their funds due to the subprime mortgage problems originated in the US[11]. This event affected other institutions along Europe, and even the Bundesbank (Germany's central bank) had to rescue national credit banks.

Starting 2008, the fear of a global recession spread around the world, and the events that took place during that year, mainly in the United States, but also in the United Kingdom and Europe, confirmed the beginning of the crisis. In fact, on January, the Large and Mid-Cap listed companies lowered their value 12.284% as an average in the five countries studied. The next few months of the year were relatively calmed for these markets. The collapse of the fourth biggest Wall Street brokerage bank – Lehman Brothers[12] – on September 15th, and the exposure of the increasing non-performing loans caused a financial shock. It drove the S&P 500 index down over 16%, while the European markets were down 10.109% in September and 14.482% in October, on average. By the end of 2008, the Italian stock market had lost half of its value, while in France, Germany and Spain the markets had lowered by 40%. In the United Kingdom, its main market index lost over 30% of its value in the same year.

Consequently, what started as a financial crisis quickly transferred into real economy around the world, due to the interdependence between the American economy and the

[11]https://www.reuters.com/article/us-bnpparibas-subprime-funds/bnp-freezes-2-2-bln-of-funds-over-subprime-idUSWEB612920070809

[12] http://www.telegraph.co.uk/finance/financialcrisis/6173145/The-collapse-of-Lehman-Brothers.html

global demand. The combination of this crisis and the imbalances present in many European countries caused that the excessive indebtedness of both families, corporations, and governments ended up collapsing and making matters worse.

The economic crisis caused world's GDP growth to be reduced to 1.82 % in 2008, while it had been growing above 4% for the past decade[13]. In 2009, the whole world entered into recession, with a negative GDP growth of 1.738% and 3.53% for OECD member countries. In global terms, GDP growth improved in the subsequent years, mostly caused by the growth of developing economies, such as China and India, while European Union countries experienced little GDP growth, even negative in 2012 for the period 2010-2015.

The recovery of the economic crisis has been characterized by the aggressive monetary policy taken from the different central banks around the globe, mainly the Bank of Japan, the Federal Reserve, the Bank of England and the European Central Bank, which applied zero-interest rate monetary policies combined with massive acquisitions of both government and corporate bonds, leading to historically low risk-free asset returns. The effects of these policies had not been determined yet in every country, though they certainly caused an artificial increase in the value of most of the developed countries' financial markets, as investors turned to riskier investments looking for new ways to obtain profits. Since 2015, the Federal Reserve started to increase rates steadily and has been followed by the Bank of England; in addition, the ECB is expected to be the next raising its rates starting in 2019 or 2020.

However, other economic events have affected the markets along this period, and there are risks that should be counted on for the future. The slowdown in the growth of China is an important factor, for example, it caused an important level of instability in the markets on August 2015[14]. In addition, the risks of a bond bubble in developed economies are an important challenge that central banks should take care of as the economist Daniel Lacalle reflects[15].

[13]https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?end=2016&locations=OE-FR-DE-IT-ES-US-GB&start=2008

[14]https://www.theguardian.com/business/live/2015/aug/24/global-stocks-sell-off-deepens-as-panic-grips-markets-live

[15] Daniel Lacalle, "Escape from the Central Bank Trap.", New York City, Business Experts Press LLC.

3.1. Eurozone during the period 2008-2017

The 19-member Eurozone or Euro area is characterized by the monetary union of the different economies under a unique central bank – the European Central Bank (ECB) - and the adoption of a single currency – the Euro. In the ten-year period considered for analysis, new countries joined the Euro, specifically, Cyprus and Malta in 2008, Slovakia in 2009, Estonia in 2011, Latvia in 2014, and Lithuania in 2015. By carrying out this economic integration process, those member countries transferred their monetary policy to the ECB, which applies a single policy that affects different countries with their own legislations and structures. In this way, each member country has its own fiscal policy, even though the Euro area is defined as a single unified market. The differences and imbalances between countries caused a great crisis in the area. Following the financial crisis of 2008 and 2009, the Eurozone bond crisis took place from 2011 to 2014, when Europe was still in a recession period and with lower growth rates than the United Kingdom or the United States. It had an important effect on the stock market of European countries compared with the United Kingdom. While in the UK, the market value of the larger listed companies had reached the pre-crisis levels in May of 2013, the France market traded at an approximate 30% discount, the German market at a 20% discount and in Spain and Italy at half of its price in January 2008.

The Eurozone crisis was a debt crisis in mainly Southern European countries, as Portugal, Spain, Italy and Greece, due to the low economic growth after the crisis and the end of the housing bubble; therefore, many households and companies were in a difficult position to repay their debts. Additionally, these economies had major economic imbalances because of internal and external deficits. The ECB had to bailout Cyprus, Portugal and Greece in this period, while the speculation of the insolvency of Italy and Spain caused instability in the European economy. However, the most critical point came in the summer of 2015 when Greece considered leaving the Euro because of the default situation that it was facing, as a result of not reaching an agreement to carry out structural reforms in exchange for credit by the ECB[16].

The performance of the president of the ECB during these years, the Italian Mario Draghi was crucial to avoid the default of Greece and keeping the cohesion of the

[16]https://www.theguardian.com/business/live/2015/jul/12/greek-debt-crisis-eu-leaders-meeting-cancelled-no-deal-live

Eurozone and encourage its members to carry out important economic reforms, as in the case of Spain. The continuous environment of low interest rates and an active bond purchasing program by the ECB also had a major importance. The combination of the active monetary policies taken from the ECB combined with the structural reforms pushed by Central Banks in different countries caused markets to stabilize first and then economies of these countries to expand.

3.2. The Spanish economy during the period 2008-2017.

The analyzed ten-year period in Spain is marked by two crises, and the following economic recovery since 2014. The Spanish crisis of 2008 was the product of both the global international financial collapse and the domestic housing crisis. Before the housing crash, the building sector in Spain represented 17.9% of GDP and employed 13% of the active population; in fact, Spain had become in 2001 the European country with the highest real estate ownership percentage[17]. The decrease in the prices of the housing market and the collapse of the demand caused a crisis, like the one started in the US. The second crisis in this period took place in 2011. This crisis was a public debt crisis in which the internal deficit in Spain reached 8.51% of GDP, and Spain was urged to take reforms. In the summer of 2012, Spain was close to being rescued by the ECB, amid a highly uncertain situation regarding its capacity to pay back its debts. Finally, this bailout only took place in the Spanish financial sector, but the government was encouraged to make structural reforms to reverse the situation. These reforms have had an important effect in the Spanish economy, that started growing in 2014, after five years of economic recession, and growing at a faster pace than the average of the European Union: 3.43% to 2.31% in 2015 and 3.27% to 1.94% in 2016.

However, the most important aspect of the Spanish economy in this period was the unemployment rate. Although historically the country already presented one of the highest structural unemployment rates, the crisis worsened the situation. In fact, starting from the lowest level of an unemployment rate of 7.93% in the second quarter of 2007, it spiked up to a record of 26.94% in the third quarter of 2013. From that point, the labor market started to recover at a slower and constant pace to the current 16.55% unemployment rate in the last quarter of 2017[18].

[17]http://www.elmundo.es/especiales/2008/10/economia/crisis2008/espana/index.html
[18]http://www.ine.es/consul/serie.do?s=EPA815&c=2&nult=100

3.3. The Italian economy during the period 2008-2017.

The evolution of the Italian economy along this period has shared some common elements with Spain even if it has its own particularities. Some of the similarities between both countries were its internal deficit, worsened significantly as a result of the crisis, an economic system based in low productive industries, and small average size of businesses. However, Italy did not experience a housing crisis similar to the Spanish one. As a result, the initial impact of the crisis in Italy was not as deep as in Spain, and the unemployment rate rose from 3.2% in 2007 to 12.5% in 2016[19], when the country was already in an expansion period.

On the other hand, the main problem for the Italian economy has been its political instability, which caused lack of strategic economic policies that could provide Italy a good starting point to grow again. In fact, even if Italy has benefited from positive global growth factors, the annual GDP growth was below 1% until 2017, when it reached 1.4%. These values are far from the European Union average and more than two percentage points below the Spanish growth.

3.4. The French economy during the period 2008-2017.

Before the start of the crisis, continuing with the global trend of liberalizing the economic system around the world, France under the continuing liberal governments undertook a process of liberalization, mainly in the banking and the automobile sector. France was one of the first countries to experience the financial crisis because their banks had major exposure to international financial institutions and the American financial sector. However, the fast intervention of the central government by implementing stimulus programs in both the financial and the industrial sector helped to endure better the crisis than other European countries. In fact, France according to World Bank's data, only suffered an economic contraction in 2009 with a decrease of the GDP of 2.94%. Although the crisis did not have a dramatic effect, as in Italy and Spain, the recovery has been slower. As a result, France should work towards having a most dynamic economy to solve a youth unemployment rate of 25% and its internal deficit (the French debt is close to 100 percent of GDP[20]).

[19]https://www.ft.com/content/fcf41e34-da57-11e7-a039-c64b1c09b482
[20]https://www.bloomberg.com/view/articles/2017-05-16/the-french-economy-is-bad-in-a-crisis

3.5. The German economy during the period 2008-2017.

Germany has been the leading economy in the European Union during the 2011-2014 Eurozone crisis and the following recovery. However, its position as the main driving force of the European economy was built in the previous two decades. After the reunification, Germany had to deal with recovering and promoting the development of the former German Democratic Republic (East Germany), in which new industries and public infrastructures were built. In addition, during the early 2000s, instead of increasing its public debt and spending in '*white elephants*' (investments, mostly public, which have a doubtful profitability) or allowing a housing bubble, Germany carried out important structural reforms. One of the most important reforms is known as Kurzabeit, which is defined by The Daily Signal as: "a federal subsidy that makes up a portion of lost pay for workers whose hours are temporarily reduced during cyclical reductions in demand. Its purpose is to encourage employers to retain trained staff so that production can recover more quickly in response to recovering demand."[21] This factor, along with the developed manufacturing and industrial sector, and which represents approximately 30% of its GDP, resulted in a positive current account balance of payments in the period 2006-2010 of 6.20%[22], in contrast to the negative values in most European countries during the period.

However, it was the economy that suffered the most in terms of GDP reduction in the start of the crisis in 2007, with a decrease of its domestic product of 5.62%, due to the demand shock experienced by its industry, and the crisis that affected many small regional banks linked with non-performing loans and debts. After this initial shock, Germany started to grow again at a faster pace mainly due to its export-driven success.[23] Consequently, Germany led the Eurozone economies in terms of growth during the period 2010-2014.

3.6. The United Kingdom economy during the period 2008-2017.

The United Kingdom is the only country analyzed that has its own independent central bank: the Bank of England. This is a relevant factor to take into account because the UK controls its own monetary policy, as well as its fiscal policy. On the other hand, the UK financial sector had more links with the US economy than European banks, which made

[21]http://dailysignal.com/2014/06/19/economic-crisis-survival-germany-shows-preparation-key/
[22]http://www.kas.de/wf/doc/kas_33244-1522-23-30.pdf?130110040302
[23]http://www.nytimes.com/2010/08/14/world/europe/14germany.html

the financial crisis worse, compared to the other European countries. In fact, the crisis in the United Kingdom began in 2007, when Northern Rock – one of most important British lending banks – had to resort for funding from the Bank of England for the first time in a century[24]. The financial crisis in the UK affected the real economy and its GDP decreased in 2008, while most other countries in Europe still had positive yearly rates of GDP. In 2009, the crisis had a major impact in the country, and its GDP decreased by 4.19%. The coordinated decisions to reduce interest rates in 2008 and 2009 by the Federal Reserve, the ECB, and the Bank of England helped the country to recover slowly, with some recession quarters in the period 2010-2012. Nonetheless, hosting the London Olympic Games in August 2012 had important positive effects for the British economy[25]. From 2013, the country experienced a more solid recovery period with an annual GDP increase over 2%. However, in June 2016, the referendum to leave the European Union and its result drove the Sterling pound down 10%, and the economy suffered a lower GDP growth than the expected in 2016. However, the mid-term and long-term effects of this decision depend on the negotiations about the new status and relationship between the UK and the European Union.

[24]https://www.telegraph.co.uk/finance/recession/4320827/UK-Recession-Timeline-of-how-the-British-economy-has-been-hit.html
[25]http://www.bbc.com/news/business-22283940

4. METHODOLOGY AND THEORETICAL MODELS USED

The objective of any type of investment is to obtain the highest possible return by assuming the lowest risk. The evaluation of how this objective has been achieved is defined as performance. Achieving the best possible performance in the mutual fund industry is the target of every manager, since those funds with a better performance attract new capital, as well as they are awarded with distinctions, as Morningstar 'stars' rating.

The performance of mutual funds is firstly reflected on their returns. The returns obtained are the amount that investors will obtain over their initial investment when they withdraw their investment from the fund. Rational investors will prefer those funds with higher returns over those with lower returns, considering a similar risk. The second metric on which performance is measured is risk. Risk is evaluated with the variance of the assets that are included in the fund. Rational investors will choose those funds that have lower risk considering a similar return.

A model in which both the overall return and risk of the investment are included is the answer to evaluate the performance of mutual funds. The first and most common model used is the Capital Asset Pricing Model (CAPM), which measures the returns above the expected result, and reflected in the Jensen's Performance Index, or Jensen's alpha.

The Capital Asset Pricing Model was introduced in the early years of the 1960's decade. It is the result of the work of the economists Jack Treynor (1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966). The CAPM is based on two principles that were already detailed: the financial market is competitive and efficient, and investors act rationally in the risk-return tradeoff. The objective of the CAPM is to establish the expected return of an investment according to the return of a risk-free investment and the risk assumed. The portfolio risk assumed according to the CAPM is divided into two different risks: unsystematic and systematic risk. The unsystematic risk is the risk that affects directly to one single firm. On the other hand, the systematic risk affects the whole market or sector and, therefore, cannot be diversified. These assumptions may seem to be unrealistic in the real world; however, the model holds in a consistent way in most cases.

The application of the CAPM to the performance evaluation of a pool of financial assets (in our case to mutual funds) is known as the Security Market Line (SML). This model

considers that the unsystematic risk has been diluted due to the diversification procedure and, therefore, only the systematic risk is analyzed. This risk is measured in a variable represented as beta (β), which relates the portfolio return with the difference between the market return and the return of the risk-free asset. Consequently, the CAPM model, collected in (1), explains that the expected return on the portfolio is the result of the risk-free asset return and the systematic risk value compounded by the difference between the market return and the risk-free asset return.

$$R_p = R_f + \beta * (R_m - R_f) \qquad (1)$$

Where: R_p is the portfolio return, R_f is the return of the risk free-asset, β represents the risk factor according to the CAPM model and R_m represents the return of the market used as a benchmark.

In this project, we carry out an analysis of a 10-year period on the monthly returns of equity mutual funds. Therefore, we have used values for both the market return and the risk-free asset in monthly terms. The market index return has been considered as the return of the MSCI large and mid-cap equity country indexes developed by MSCI[26] in the five countries analyzed. The risk-free assets are the monthly expected returns over the 90-day Interbank rates in the Eurozone and the United Kingdom. That is, the Interbank rate is the same for every Eurozone country, while in the case of the United Kingdom, the interbank rate is set by its own authority.

From the CAPM, Michael Jensen introduced his factor in 1968, known as Jensen's alpha (α), which represents the return obtained over the expected result; that is, the value added by managers over the market and risk conditions, collected in (2). Therefore, investors will look for those funds that present higher levels of this value in a consistent basis, even if there is not proof that previously good performing funds will keep giving consistent returns over time.

$$\alpha = R_P - [R_f + \beta * (R_m - R_f)] \qquad (2)$$

The original CAPM model simplifies the expected return of a portfolio to two variables, while the financial market is more complex. As a result, other models have extended the original CAPM to include other factors. One of these models is the Fama and French three-factor model (1993), which besides systematic risk also includes the difference

[26]www.msci.com

between small and large-cap stocks and the small to high book-to-market ratio to examine the importance of the size of the companies in which the portfolio is investing as well as the book-to-market ratios of these companies to the portfolio return. Subsequently, Carhart (1997) adds the momentum factor, creating the known four-factor model. Nonetheless, the CAPM results confirm the validity of this model, despite of its simplicity, and the CAPM is one of the most used models in finance, so this work is focused on the alpha of Jensen obtained from the CAPM.

4.1. Methodology of the performance fund analysis: The optimal diversification according to Markowitz.

The analysis and comparison of the active funds on the ten-year period analyzed is performed by using the Capital Asset Pricing Model. After obtaining the best performing funds (those with a higher Jensen's alpha), we then test the optimization degree of these funds in terms of profitability and risks.

The analysis is carried out by testing the return maximization and the risk minimization of the funds to check if they are efficient according to the diversification model proposed by Markowitz. Markowitz explains that managers may maximize the portfolio return, given a risk level, or minimize the portfolio risk, given a return level. The profitability or return of a portfolio is obtained by compounding the weight that each financial asset held by the fund times its own expected profitability; that is, the weighted average return of the stocks held by the fund as represented in (3). The analysis of a portfolio risk is carried out by analyzing the variance (σ^2) by compounding matrixes with the first one being the transposed weight of the assets, multiplied by a quadratic matrix of variance and covariances of every stock with the others, and the weight of the assets, as represented in (4).

$$Return = w_1 \times r_1 + w_2 \times r_2 + \cdots + w_n \times r_n \qquad (3)$$

$$\sigma_p^2 = (w_1 \quad w_2 \quad \dots \quad w_n) \begin{pmatrix} \sigma_1^2 & \sigma_{1,2} & \dots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & \dots & \sigma_{2,n} \\ \dots & \dots. & \dots & \dots \\ \sigma_{n,1} & \sigma_{n,2} & \dots & \sigma_n^2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{pmatrix} \qquad (4)$$

Where: r_i is the return of each individual stock, w_i is the weight of each fund stock that this model calculates, $\sigma^2{}_P$ represents the variance of the fund, σ_i^2 is the variance of each fund stock, and $\sigma_{i,j}$ is the covariance between each pair of stocks held by the fund; that is, stocks *i* and *j*.

We then analyze whether the funds studied achieved the objective of maximizing profit or minimizing risk. In this analysis we compare the actual weights of each stock owned by the funds with the optimal weight that it should have in the optimal fund holding (maximizing return or minimizing risk). The testing is done with two different programs for each country fund and period analyzed. First, the objective is to maximize the profitability of the fund by designing a fund with the same or lower risk of the original fund holding, as represented in (5). The second test is to minimize the risk of the fund with the same or higher return as in the original fund holding, as detailed in (6). These methods have been used both in the calculation of the original portfolio expected returns and risks and on the optimal portfolio that is obtained after.

$$Max_{Return} \tag{5}$$

Subject to:

$$\sigma^2_{Opt} \leq \sigma^2_{Org}$$

$$\sum(w_1 + w_2 + \cdots + w_n)_{Opt} = \sum(w_1 + w_2 + \cdots + w_n)_{Org}$$

$$w_1, w_2, \ldots w_n \geq 0$$

Where: σ^2_{Opt} is the variance of the fund returns obtained with the optimal weights, and σ^2_{Org} is the variance of the fund returns obtained with the original fund weights.

There are some constraints imposed in the possible values that optimal weights can obtain. The first restriction is that we consider the variance of the optimal portfolio to be the same of the original one. The second restriction is that the sum of the stocks that form the optimal portfolio should be equal to the sum of the original weights. The second restriction does not follow Markowitz's model because originally the fund should be composed totally by stocks and, therefore, the sum of the weight of those stocks should equal to one. However, we consider this to present comparable results with the actual situation, and the total sum of stocks both in the original and the optimal fund should be the same. The last restriction represents that weights had to be higher or equal to zero, in the way that short positions cannot be taken.

$$Min_{risk} = Min_{\sigma^2} \tag{6}$$

Subject to:

$$R_{Opt} \geq R_{Org}$$

$$\sum(w_1 + w_2 + \cdots + w_n)_{Opt} = \sum(w_1 + w_2 + \cdots + w_n)_{Org}$$

$$w_1, w_2, \ldots w_n \geq 0$$

Where: R_{Opt} is the return of the fund with the optimal weights, and R_{Org} is the return of the fund with the original weights.

In the case of minimizing the risk, the only different constraint with respect to the return maximizing problem is that the return desired, and finally obtained, is the same or higher to the original portfolio return (the variable studied in this case is the variance of the portfolio).

There is an important limit that should be acknowledged in the estimation of the efficiency of the funds. Mutual fund managers pick their stocks according to different strategies, but it is obvious that they do it without knowing the future expected results. However, when we run the tests to obtain if the holding is optimal, we already know the daily return of each asset. As a result, it is statistically impossible to obtain a portfolio that is perfectly efficient with respect to maximizing profit or reducing risk in the Markowitz's strict sense. Then, we examine the degree to which mutual fund managers deviate from the optimal composition, and how their efficiency progresses over time, or if it is just a random result.

To this objective, we use the top performing mutual funds in each country with data for at least 12 months and that are active during the year 2017. We carry out this analysis for all quarters in 2017.

5. PERFORMANCE OF MUTUAL FUNDS ANALYSIS

The analysis of mutual funds results is the first researching part of this project. We apply the performance theory of mutual funds (CAPM alpha) in the study of mutual funds. The analysis of mutual funds in the five countries analyzed is performed in the period between January 2008 and December 2017 for a total of 120 months. The analysis is performed in a national level describing the most relevant results overall, and then focusing on the best performing mutual funds and ranking them. The funds that have the best alphas in each country and are active in the year 2017 are the funds selected for their composition analysis. This section concludes with a comparison between the analyzed countries to detect the existence of differences between each country managers.

5.1. Description of the data sources.

The mutual fund data is comprised by more than 3,500 mutual funds for these five countries from January 2008 to December 2017. We must point out that most of the funds are not operative in the whole period of analysis since some of them closed and others appear throughout the period. The inclusion of all funds avoids the called survivorship bias. Data of mutual funds return in a monthly basis and portfolio holdings in a quarterly basis are provided by Morningstar[27]. Data of the considered in this project as the risk-free asset is obtained from the Federal Reserve Bank of Saint Louis data base. The evolution of the price of the shares owned by all funds in 2017 is obtained on daily basis from Yahoo! Finance and Datastream.

5.2. Mutual funds results of Spain.

As it was already explained in section 2.1, Spain is the selected country with a lower level of investment in mutual funds. We may highlight the low number of Spanish funds with available information compared to the United Kingdom or France, while it is similar to the number of funds in Italy. The total number of funds that were active more than one month in the period 2008-2017 is 249. The average active months of mutual funds in Spain during this period are 62.3.

The result of the difference between market return and risk-free assets (calculated as a factor of the CAPM model) was, on average, -0.394% per month. On the other hand, the monthly average mutual fund return is -0.155 %. Even if this is a negative result, the

[27] http://www.morningstar.es/es/research/funds

return is better than the expected market result, so it should result in a positive average alpha. Specifically, the monthly average fund alpha result is 0.230 %.

We also evaluate the performance of each individual mutual fund and order them in terms of their performance according to Jensen's factor (alpha). Table 1 shows the best performing mutual funds in Spain with an operating life higher than 12 months.. The best mutual fund consistently is *QMC II Iberian Capital D FIL* that along a 56-month period had consistent overachieving results. Its monthly average return is 1.732%, with a significant 1.265% alpha factor that shows a marked improvement over the expected returns. However, since there was not information available about the portfolio holdings of this fund we analyze the performance of the next best performing fund: *azValor Iberia FI* in section 6.1. The average monthly return of this fund is 1.243% during the 26 months of its activity (from November 2015 until the end of 2017), with an alpha of 1.217%, which is very close to the previously deemed as the best consistently performing fund.

Table 1. Ranking of Spanish Mutual Funds from 2008-2017

Rank	*Name of the fund (number of activity periods)*	*Average return*	*Average standard deviation*	*Average alpha*
#1	QMC II Iberian Capital D FIL (53)	1.732 %	5.324 %	1.265 %
#2	azValor Iberia FI (26)	1.243 %	3.375 %	1.217 %
#3	Santander Small Caps España... (21)	1.803 %	3.740 %	1.216 %
#4	Magallanes Iberian Equity P FI (34)	1.100 %	3.322 %	1.175 %
#5	QMC Iberian Capital A FIL (53)	1.611 %	4.961 %	1.173 %

5.3. Mutual funds results of Italy.

Italy is the other Mediterranean country analyzed in this project. Therefore, its data will be especially comparable with the results previously analyzed for Spain. In addition, Italy is the country with the second lowest mutual funds capital of the group. The number of active funds during this period analyzed with available data is 247 funds. Their average operative life is 52.1 months. This result shows that Italian funds have a

lower operating life in this period, and a larger number of them were opened and closed along the ten years studied.

If we analyze the results of the Italian market in this period with the excess market return over the risk-free asset return, we obtain an average of -0.595%, which is worse than in Spain. The average monthly return of the funds of -0.389% is also lower than in the case of Spain. Since the return of mutual funds is higher than their expected market return, we can guess that the average alpha of these funds will be positive. Actually, the average alpha result for Italian mutual funds is 0.408%.

The individual evaluation of each mutual fund analyzed gives us the best managed funds during this period. In the Italian case, it is noteworthy that the six funds with a higher alpha were active for only two or three months during the period January 2008-March 2008. Those results are not considered to be representative, and we select the 7th best fund in order to analyze a well-managed fund during a larger period of time. In Table 2, the five best performing Italian funds operative for more than 12 months are presented. The best fund is *Symphonia Azionario Small Cap Italia I,* which has been operative for 23 months. Its first active period was February 2016, and it has continued operating with an average monthly return of 1.948%. Its return has exceeded the expected market return by 1.621%, which is an alpha result higher than the selected as the best performing Spanish fund in this period. The composition and return-risk optimization of this fund will be analyzed in section 6.2.

Table 2. Ranking of Italian Mutual Funds (2008-2017)

Rank	*Name of the fund (number of activity periods)*	*Average return*	*Average standard deviation*	*Average alpha*
#1	Symphonia Azionaro Small... (23)	1.948 %	3.773 %	1.621 %
#2	ATOMO Made in Italy I EUR Acc (19)	2.191 %	4.741 %	1.583 %
#3	ATOMO Made in Italy R EUR Acc (19)	2.161 %	4.714 %	1.567 %
#4	ATOMO Made in Italy L EUR Acc (15)	2.759 %	5.148 %	1.354 %
#5	Arca Economia Reale Equity Italia I (32)	1.227 %	3.916 %	1.342 %

5.4. Mutual funds results of France.

France is the first continental European country analyzed. France is one of the most developed countries in the world in terms of financial asset management, being the first continental European country in this aspect. The total capital invested in financial assets, and especially mutual funds, is higher than in the previous cases of Italy and Spain. The number of mutual funds that were operative for more than two months is 504 mutual funds. The average length of their activity fund period is 82.8 months. These results show that, in addition to the large amount of registered French mutual funds, their period of activity is longer than in the other analyzed European countries.

The analysis of the performance of the French stock market and mutual funds provides different results with respect to the previously analyzed countries. The monthly average return of the market minus the risk-free asset is also negative (-0.123%) due to the negative situation in the first years of the period. The monthly performance of French mutual funds, conversely to Spanish or Italian ones, had a positive monthly average return of 0.466%, which is a very good result for French investors. Finally, the average value of alpha is 0.383%.

If we analyze the individual performance of French mutual funds in this period, we find that the best two funds were active only for two months. Following these two mutual funds, we find three funds from the asset management firm *Kirao*, as presented in Table 3. This table only represents those funds active for more than 12 months. However, data about their portfolio holdings were not available. Therefore, the sixth best managed mutual fund is analyzed in section 6.3. This fund, called *ID France Smidcaps I*, has been active for 57 months, which is a significant amount of time to consider. This fund has consistently obtained better results than the market, presenting a monthly average return of 2.157% with an alpha of 1.787%. In addition, it is also remarkable the stability of these returns, since the standard deviation of the returns is lower than in most of the other best French mutual funds.

Table 3. Ranking of French Mutual Funds (2008-2017)

Rank	*Name of the fund (number of activity periods)*	*Average return*	*Average standard deviation*	*Average alpha*

#1	Kirao Smallcaps NC (32)	2.296 %	3.239 %	2.181 %
#2	Kirao Smallcaps IC (32)	1.979 %	3.244 %	1.853 %
#3	Kirao Ristretto AC (30)	1.996 %	3.639 %	1.806 %
#4	ID France Smidcaps I (57)	2.157 %	2.877 %	1.787 %
#5	Kirao Smallcaps AC (32)	1.905 %	3.221 %	1.780 %

5.5. Mutual funds results of Germany.

Germany is the second continental European country analyzed. As it has already been explained in section 2.4, German investors present a conservative attitude towards taking risks and participating in the financial market. The total capital invested in mutual funds is higher than in the Mediterranean countries analyzed, but lower than in the European leader (United Kingdom) and the other continental European country studied: France. The available analyzed information includes 101 German mutual funds. The average operating life of these funds analyzed stood at 69.3 months.

The performance of the German markets and mutual funds shows that the difference between the stock market and the risk-free asset return is, on average, -0.008%, which is the best return in all the cases analyzed (less negative monthly return). In the case of the mutual fund performance, the average monthly return was 0.138% for all funds. The value of Jensen's factor (alpha) is 0.420%, which is the second highest result, only behind British funds.

If we focus on the performance of individual mutual funds, we find results similar to the previously analyzed countries. The three best performing funds were active for less than a year. In Table 4, we present the best five performing funds with an operating life longer than one year. The best performing mutual fund active for more than one year, and operative in the year 2017, is the seventh best performing fund overall. This fund, called *BERENBERG-1590-Aktien Mittelstand I*, has been active since January 2016. Its monthly average return is 1.648%, with a monthly value of alpha of 1.133%, the lower result of the selected best performing country funds. We must also point out the low standard deviation value of their returns, which is the second lowest of the top 12 funds. This characteristic is also valued by investors because they look for the safest ways to obtain consistent returns.

Table 4. Ranking of German Mutual Funds (2008-2017)

Rank	*Name of the fund (number of activity periods)*	*Average return*	*Average standard deviation*	*Average alpha*
#1	DWS Small & MidCap Growth... (28)	-0.213 %	10.621 %	1.318 %
#2	DWS Zürich Invest Atkien... (23)	-0.999 %	9.777 %	1.212 %
#3	Cominvest Incofonds (21)	-1.140 %	10.060 %	1.152 %
#4	BERENBERG-1950-Aktien... (24)	1.648 %	4.271 %	1.133 %
#5	Allianz RCM Adiselekt P (40)	0.712 %	7.971 %	1.067 %

5.6. Mutual funds results of the United Kingdom

The United Kingdom is the country in which many of the big worldwide financial firms are located. British investors are those with a largest per capita investment in financial assets in the European countries analyzed. In addition, the total capital invested in mutual funds is the second largest of the group, behind France. We analyze 2,577 UK domestic equity mutual funds. The average operating activity of these funds stands at 78.7 months.

The monthly average difference between the market return and the risk-free asset considered is negative, as in all the previous cases, at -0.049%. The average monthly return of the funds is 0.576%, which is the highest value of all the countries analyzed. The average alpha is also the highest of the group, standing at 0.465%.

In Table 5 we analyze the best five performing funds with an operating life larger than 12 months. The selected fund for the analysis of its composition and its performance in terms of maximizing returns and minimizing risk is the sixth best fund in terms of the Jensen's performance measure. This fund is *Old Mutual UK Smlr Coms Foc U1 GBP Inc* and is the longest active of the ranked top five best funds, with 29 active months since its creation in August 2015. The average monthly return provided by this fund is 2.469%, with an alpha of 2.234%. This value of alpha is the highest of the selected funds in all countries. The main reason for this is the existence of a larger number of

funds in the UK, compared to the other countries, as the worst performing funds are also in this country. The standard deviation of its returns is also significant, standing at 4.326%, which is the second highest variability value in the top 10 of the best performing funds.

Table 5. Ranking of UK Mutual Funds (2008-2017)

Rank	*Name of the fund (number of activity periods)*	*Average return*	*Average standard deviation*	*Average alpha*
#1	MI Chelverton UK Equity Growth... (18)	2.902 %	2.696 %	2.477 %
#2	Old Mutual UK Smlr Coms Foc... (29)	2.469 %	4.326 %	2.234 %
#3	Old Mutual UK Smaller Coms... (13)	2.137 %	3.551 %	1.809 %
#4	EFA OPM UK Equity A Inc (14)	2.525 %	2.928 %	1.809 %
#5	Schroder UK Alpha Income A... (17)	-1.168 %	8.562 %	1.800 %

5.7. Comparison of these results.

After analyzing the results of each country, we compare some of the most significant values found. Part of these results are already explained in the corresponding country analysis. Table 6 shows this summary.

Table 6. Results of mutual funds according to their origin country (2008-2017)

	Spain	**Italy**	**France**	**Germany**	**U.K.**
Number of funds analyzed	249	247	504	101	2577
Avg. life of the funds (months)	62.3	52.1	82.8	69.3	78.7
Avg. $R_m - R_f$	-0.394 %	-0.595 %	-0.123 %	-0.008 %	-0.049 %
Avg. Return	-0.155 %	-0.389 %	0.466 %	0.138 %	0.576 %
Avg. Alpha	0.230 %	0.408 %	0.383 %	0.420 %	0.465 %
Percentage funds with positive alpha	81.93 %	88.26 %	91.27 %	91.09 %	94.57 %

According to the previous table, we obtain some conclusions about the performance of mutual funds in the ten-year period analyzed for each country. The United Kingdom is the country in which both the average monthly return of mutual funds and the average alpha are the highest, as well as the percentage of mutual funds with a positive alpha value. These results display that the United Kingdom mutual managers as the best of their class, in average. As a result, it should come as no surprise that the United Kingdom is the country with the largest number of mutual funds and the biggest financial industry. In the case of Germany and France, both countries have similar results, while the former leads in average return and the latter in the average alpha. Among these two countries, Germany is the one in which its market had better results during the period. However, French mutual funds obtained a higher average monthly return, even if their average alpha is lower than in German funds. On the other hand, Mediterranean countries (Italy and Spain) have worse performance than Continental European managers (France and Germany). The average monthly performance of their market is worse in both countries, as explained in sections 3.2 and 3.3. The value of alpha in Italy is the third highest of the five countries, but its low market return caused the lowest average monthly return in Italian funds. Spanish funds had a lower value of alpha and provided also negative average monthly returns to their investors. Therefore, UK mutual fund managers obtain the best results, followed by French and German mutual fund managers, while Spanish and Italian managers are the worse of the group. We must point out that this analysis is carried out in a specific ten-year period and investment location category; therefore, results are limited to this period and category and cannot be generalized to a larger period, the future, or other categories.

6. MUTUAL FUND PERFORMANCE ASSESSMENT.

In the last section of the project we analyze the selected as best performing mutual funds in each of the countries considered and check if these funds are optimal in terms of profit or risk according to Markowitz. This analysis is performed by using the models explained in section 4.2. of the project with regard to the return and variance of the funds. The study is performed individually over the selected mutual funds quarterly during the year 2017. The analysis is performed in two stages: first, we analyze the individual composition (portfolio holdings) of each mutual fund for the fourth quarter of 2017 and compare the actual shares owned by the portfolio with the optimal composition. Second, we compare the results and evolution of return and risk optimization in the four quarters of 2017 analyzing if managers change their portfolio holding strategies over time. The stock price data used is presented on a daily basis for each of the individual stocks that the mutual fund owns during the period. Therefore, the best solution to interpret these data is the relative comparison of optimal results with real results. We must point out that it is statistically impossible that a mutual fund that owns a large number of different company shares can pick the specific optimum weights for each of the companies ex-ante. Therefore, we are not analyzing perfection in each of the funds but how - being the best performing mutual funds in their respective countries - accurate the managers of these funds are.

6.1. Optimization of the best performing Spanish mutual fund.

The selected Spanish mutual fund to analyze its optimization is *azValor Iberia FI*, with an average monthly return in 2017 of 1.593% and a standard deviation of 2.29%. After selecting this fund as the best performer in Spain during this period, we analyze its composition to study if the mutual fund optimizes its return and/or risk. First, we compare the actual composition of the fund with the optimal composition that would maximize the return or minimize the risk during the fourth quarter of 2017. We then compare the optimization degree in the remaining quarters of 2017.

During the fourth quarter of 2017, the selected mutual fund obtained a daily average return of 0.0722%, with a daily variance of 0.00212%. As it can be observed, both values are very low, but they are the result of using daily data. In the case of total return, it shows that the total return obtained along the period is positive. The value of the fund variance is very low due to the large number of stocks held. The larger the number of stocks, the risk of the investments tends to disappear. During this quarter, the mutual

fund selected owned a total of 27 stocks. As part of those stocks, we highlight the five stocks with the largest weights over the total portfolio (to later compare them with the optimal result). These stocks are: *Galp Energía SGPS SA* (8.37%), *Elecnor SA* (7.94%), *Tubacex SA* (7.77%), *Técnicas Reunidas SA* (6.33%), and *Mota-Engil* (4.79%).

The next step is to check for their possible optimization. In the case of return maximization, we consider that the risk assumed of the portfolio is the same as in the real fund. The maximization result shows a daily average return of 0.296%, which is 0.224% higher than the actual return. In this case, the portfolio would be composed by 23 stocks. The optimal return portfolio with the corresponding shares, their daily average return and standard deviation are represented in Table A.3 of the Annex.

In the case of risk minimization, we consider a portfolio with the same daily average return as the real fund return. In this case, the optimal risk is 2.7 times lower than the current risk. The portfolio variance is 0.000784 %. The optimal portfolio risk with the portfolio holdings, average return and standard deviation are represented in Table A.4 of the Annex.

If we analyze the optimization results in the other quarters, we obtain some differences. During the third quarter, the average daily return was negative (-0.012%), and the optimal return is 0.135%, a positive difference of 0.148% with the real return, while the optimal risk is 2.5 times lower than the actual risk in this period. In the second quarter, the optimal return stood at 0.419%, which is an average of 0.293% higher than the actual return (0.127%). In this same period, the optimal risk is 4.38 times lower than the actual risk, being this period the one with a larger difference in the case of both optimal results and, therefore, the period in which the manager missed the most in her selections. For the first quarter of the year, there is a positive difference between the optimal return (0.266%) and the current return (0.079%) stands at 0.187%, while the optimal risk is 3.78 times lower than the actual variance. These results are presented in a synthesized form in Table A.5 of the Annex.

6.2. Optimization of the best performing Italian mutual fund.

The selected Italian fund for the analysis of its optimization result is *Symphonia Azionario Small Cap Italia I*. This mutual fund had a monthly average return of 2.641% in 2017, which is higher than the best performing Spanish mutual fund. The standard deviation is 0.04, which is also higher than in the Spanish fund. As in the previous case,

we analyze the optimization in the fourth quarter in detail and point out the results in the other quarters of the year.

In the fourth quarter of 2017, the selected mutual fund obtained a negative daily return of -0.007%, the only quarter with negative return. The risk of this portfolio is 0.0035%, which is also low due to the diversification strategy. In this case, the fund was composed in 92.03% by shares, while the rest were bonds and cash. The total number of shares that compose the fund are 78, and only 25 shares represent a percentage over 1% of the total portfolio. The five most significant stocks are: *Interpump Group SpA* (7.17%), *BB Biotech AG Ord* (6.18%), *Amplifon SpA* (5.25%), *Industria Macchine Automatiche SpA* (4.48%), and *EI Towers SpA* (3.45%).

The analyses of maximization return, and minimization risk are especially noteworthy as the top five mentioned stocks had a very low ownership share in both optimal cases. In the maximization return, the optimal result shows a return of 0.251% with a difference between the actual return (-0.007%) and the optimal result of 0.258%. In this case, the portfolio would only be composed by 14 stocks in comparison with the 78 stocks of the original portfolio. The return portfolio composition is shown with its individual shares return and standard deviation in Table A.8 of the Annex.

In the case of minimizing variance, the optimal result is 3.25 times lower than the actual variance of the portfolio. The number of shares that would compose this portfolio is also lower than in the original portfolio but higher than in the profit maximization case, as it stands at 28 stocks. The three stocks with a higher weight are represented in Table A.9 of the Annex.

The analysis and comparison of the optimization results for the rest of the quarters of 2017 helps in assessing the performance of the fund manager and shows if the manager is closer to obtain a maximum return or minimum risk. In the third quarter, the optimal return is 0.227% higher than the actual portfolio return (0.186%), with an optimal daily return of 0.413%. In the case of the variance, the result is very similar to the one obtained in the fourth quarter, as the optimal variance is 3.65 times lower than the actual result. For the second quarter, the optimal return is 0.472%, which represents a spread with respect to the actual return (0.073%) of 0.399%, higher than in any other period. The optimal variance is also the furthest from the actual case, 5.45 times lower than the actual portfolio. In the first quarter, the optimal return is 0.217% higher than the actual result (0.307%) and stands at 0.523%. The optimal variance is 2.62 times lower than the

original portfolio. As we can observe, the most optimal results are achieved in the first quarter, while the results are the furthest from the optimal cases in the second quarter. These results are presented in a synthesized way in Table A.10 of the Annex.

6.3. Optimization of the best performing French mutual fund.

In the case of France, we analyze the fourth best performing mutual fund: *ID France Smidcaps I*. The monthly average return of this fund stands at 2.216% while the standard deviation is 2.568% in 2017.

During the fourth quarter of 2017, the fund obtained a daily average return of 0.06%. The variance and as a result the risk is lower than in the previous cases of Italy and Spain, as it is 0.00184%. Additionally, 97.17% of the total portfolio holdings are in stocks. The number of stocks owned is 47. The main characteristic of this fund is the similar weight among the stocks, with individual percentages lower than 3%. The five stocks with a higher representativeness are: *Esi Group SA* (2.52%), *Fountaine Pajot SA* (2.50%), *Groupe Open SA* (2.44%), *Trigano SA* (2.44%) and *Pharmagest Interactive* (2.42%).

The optimal return with the maximization program is 0.271%, which is a positive difference over the result of the real portfolio of 0.21%. The risk of this portfolio would be the same as in the real fund, but only 20 shares would be part of this portfolio. In this case, the optimal profit portfolio for the fourth quarter is presented in Table A.13 of the Annex.

In the analysis for the optimal variance, the optimal result is five times lower than the real fund risk. Therefore, the manager had worse results in terms of risk compared with the Spanish and Italian best performing managers in this period. The portfolio that represents the optimal minimizing risk is displayed in Table A.14 of the Annex.

In the third quarter of 2017, we obtain that the optimal daily average return (0.102%) is only 0.042% higher than the real return (0.059%), which is the closest result found throughout this whole analysis. In this period, the optimal variance is ten times lower than the actual variance. These results show that the manager of this fund achieves a better result with regard to the optimal returns compared with the Italian or Spanish best performing funds. In the second quarter of 2017, the positive difference between the optimal return (0.431%) and the actual return (0.179%) was 0.252%, which is a difference similar to the one obtained in the rest of the cases. In terms of the variance,

the optimal risk is 3.7 times lower than the real risk. In the first quarter, the optimal return (0.277%) is 0.132% higher than the real return (0.145%), while the optimal variance is five times lower than in the originally selected portfolio. As we can observe, in three of the four quarters analyzed, the difference between the optimal and the actual return is closer than in the selected Italian and Spanish mutual fund, and the variance of the portfolio is larger than in those cases. These results are presented summarized in Table A.15 of the Annex.

6.4. Optimization of the best performing German mutual fund.

The best performing mutual fund analyzed in Germany is called *BERENBERG-1590-Aktien Mittelstand I*. During 2017, the selected mutual fund has a monthly average return of 2.883%, which is higher than the return of mutual funds in France, Italy or Spain in this period, and the standard deviation is 2.7489%.

In the fourth quarter of 2017, the daily average return is 0.051% with a variance of 0.00382%. This mutual fund was composed in 92.26% by equities with 44 different shares in this portfolio. Similar to the French case, the weight of each share is not higher than 3.50%. The five stocks with a higher weight are: *Wirecard AG* (3.50%), *Stabilus SA* (3.11%), *Patrizia Immobilien AG* (3.08%), *Zalando SE* (3.07%), and *Grenkeleasing AG* (3.04%). Those are the only shares in the fourth quarter of 2017 with a fund weight higher than 3%.

If we analyze the optimal solutions, the optimal return in this quarter is 0.228%, 0.177% higher than the real return. In this case, only ten shares of the original portfolio would compose the optimal one. The optimal return and standard deviation of the shares held is presented in Table A.18 of the Annex.

The optimal minimizing variance portfolio is 2.65 times lower than the original result. This portfolio would be composed by 15 different shares. The optimal risk portfolio with their portfolio holdings as displayed in Table A.19 of the Annex.

We also analyze the other quarters of 2017. In the third quarter, the optimal return (0.338%) is 0.174% higher than real return (0.164%), while the optimal variance is 2.85 times lower than the original portfolio variance. These results are similar to the ones obtained in the fourth quarter. During the second quarter, the difference between the optimal (0.332%) and the real return (0.123%) is 0.209%, which is the largest difference in the year. In this period, the variance of the optimal portfolio is 1.9 times lower than

the variance of the original portfolio, which is the best result in any period of the five mutual funds analyzed. In the first quarter, the difference between the optimal return (0.316%) and the real return (0.174%) is 0.143%, which is the lowest of the year. On the other hand, the optimal portfolio variance is 2.79 times lower than the original portfolio variance. The management of this fund has obtained positive results in terms of both profit and variance in 2017, and the difference with the optimal portfolios are some of the lowest, which indicates a continuous good performance by the managers of this fund. These results are presented in a synthesized way in Table A.20 of the Annex.

6.5. Optimization of the best performing United Kingdom mutual fund.

The best performing mutual fund in the United Kingdom during the ten-year period analyzed is *Old Mutual UK Smlr Coms Foc U1 GBP Inc*. During 2017, the average monthly return is 3.477%, which is the best fund result of the five mutual funds analyzed. The standard deviation of this twelve-month period portfolio is 2.319%.

In the fourth quarter of 2017, the daily fund return is 0.131% with a variance of 0.00418%. The portfolio was composed in 93.90% of equity and with 54 different stocks. The five stocks with the largest portfolio weights are: *Blue Prism* Group *PLC* (6.33%), *Fevertree Drinks PLC* (5.37%), *Microgren PLC* (4.00%), *Alpha FX Group PLC* (3.81%), and *Ascential PLC* (3.69%).

First, we analyze the maximization return. The optimal average daily return is 0.548%, which supposes an increase over the original portfolio return of 0.417%. In this portfolio, only 14 of the original shares would compose this optimal fund. The optimal return portfolio with the weights of each share are presented in Table A.23 of the Annex.

If we analyze the risk optimization, the optimal variance is 2.49 times lower than the original portfolio variance. The optimal risk portfolio composed by 16 different shares is represented in Table A.24 of the Annex.

In the third quarter, the difference between the optimal (0.418%) and the original return (0.208%) is 0.210%, while the optimal variance is 3.38 times lower than the original result. In the second quarter, the difference between the optimal (0.540%) and the original return (0.274%) stands at 0.256%, while the variance of the optimal portfolio is 4.31 times lower than in the original case. For the first period, the fund return in terms of maximizing the profit is the best result of the year, as the difference between the

optimal (0.502%), and the original portfolio (0.297%) return is 0.206%. The variance of the optimal portfolio is 3.95 times lower than the original portfolio risk. This mutual fund obtains results in line with the other mutual funds studied; however, in the difference between the original and the optimal variance, the fund shows consistently higher results than in the other analyzed mutual funds. The optimal and the actual results for the four quarters of 2017 are represented in Table A.25 of the Annex.

We should note that these analyses represent an after-the-fact research, and, as it was pointed out in the introduction of this section, it is statistically impossible to achieve the optimal situation. Therefore, although there is a certain spread between the actual and the optimal results, we cannot conclude that it was an inadequate management of these funds. On the other hand, these funds are the best performers in their countries, and the difference between the real and the optimal results is very small in many cases. Furthermore, we observe that the experience of the managers provides an extra positive value to the investor that they would not achieve by replicating the market or choosing other investment instruments.

7. CONCLUSIONS OF THE PROJECT

In this study, we have analyzed mutual funds as an investment instrument available to institutional and individual investors. We have studied the different types of investment funds and focused in the domestic equity mutual funds of five European countries: Spain, Italy, France, Germany and the United Kingdom. The objective of selecting five countries was to show the state of the financial industry in these countries, the conditions under which mutual funds are subject to and their evolution in the last ten years. We have also studied the macroeconomic and stock market situation in these countries.

In the second part of the project we have analyzed actual data about mutual funds from these countries. Firstly, we have analyzed their performance with the CAPM model. We have also established the optimization problem under which we have studied if the managers of the best performing funds obtain maximum return or minimum risk portfolios. After explaining the theoretical models, we have analyzed more than 3,000 mutual funds in the countries selected, establishing those with higher values of alpha. The main conclusion is that the majority of mutual funds obtained a positive alpha due to the knowledge and expertise of professional managers. On the last part of the project, we have focused on assessing the optimal returns and variances of the best performing portfolios. From this analysis, we find that mutual funds are capable of being consistently close to the optimal return or the optimal variance, especially in the selected German mutual fund.

We can conclude that managers from northern European countries (France, Germany and the United Kingdom) obtain better results than managers of southern European countries (Italy and Spain). However, the differences of all optimal results and the performance of these managers are similar in most cases. Therefore, the main differences among those mutual funds come from their specific national macroeconomic situations, which affect the performance of their financial market. As a result, the conclusion from an investor point of view is that investors should analyze and detect the trends and state of each market and economy before investing in equity mutual funds because this will be the main influence in the mutual fund return.

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ANNEX

Table A.1. azValor Iberia FI portfolio composition, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.
GALP	Galp Energia SGPS SA	8,37%	-0,027%	0,009413
ENO	Elecnor SA	7,94%	0,066%	0,008977
TUB	Tubacex SA	7,77%	0,058%	0,015463
TRE	Tecnicas Reunidas SA	6,33%	-0,043%	0,010924
EGL	Mota-Engil	4,79%	0,239%	0,018889
NOS	Nos Sgps SA	4,67%	-0,179%	0,013128
SNC	Sonaecom SA	4,64%	-0,230%	0,037984
CCE	Coca-Cola European Partners PLC	4,19%	0,036%	0,006033
ZOT	Zardoya Otis SA	4,02%	0,088%	0,012435
MEL	Melia Hotels International SA	4,02%	0,185%	0,01226
EKT	Euskaltel SA	3,97%	0,185%	0,012104
TEF	Telefonica SA	3,51%	0,234%	0,009743
ALM	Almirall SA	3,09%	0,039%	0,012409
SONC	Sonae Capital SGPS SA	2,95%	0,166%	0,014412
MCM	Miquel y Costas y Miquel SA	2,92%	0,080%	0,016414
GCO	Grupo Catalana Occidente SA	2,82%	0,128%	0,010173
FCC	Fomento de Construcciones y Contratas SA	2,78%	0,182%	0,013398
IBS	Ibersol SGPS SA	2,43%	0,289%	0,021396
ACX	Acerinox SA	2,24%	0,086%	0,016492
JMT	Jeronimo Martins SGPS SA	2,11%	0,057%	0,011573
MAS	Masmovil Ibercom SA	2,07%	0,708%	0,014907
GAS	Gas Natural SDG SA	1,81%	0,203%	0,008594
VIS	Viscofan SA	1,53%	-0,018%	0,009045
IPR	IMPRESA SGPS SA	0,73%	0,253%	0,029922
SEM	Semapa SA	0,61%	0,074%	0,011391
OLE	Deoleo SA	0,35%	-0,296%	0,037755
CMO	Cementos Molins SA	0,30%	0,130%	0,014507
TOTAL		92,96%		

Table A.2. Results of azValor Iberia FI, 4th quarter of 2017

Average daily return of the portfolio	0.0722%
Variance of the portfolio	0.0000213

Table A.3. azValor Iberia FI Optimal Return Portfolio Results, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
TUB	Tubacex SA	7,77%	0,058%	0,015463	1,91%
TRE	Tecnicas Reunidas SA	6,33%	-0,043%	0,010924	0,05%
EGL	Mota-Engil	4,79%	0,239%	0,018889	4,92%
NOS	Nos Sgps SA	4,67%	-0,179%	0,013128	0,03%
CCE	Coca-Cola European Partners PLC	4,19%	0,036%	0,006033	9,90%
ZOT	Zardoya Otis SA	4,02%	0,088%	0,012435	0,05%
MEL	Melia Hotels International SA	4,02%	0,185%	0,01226	12,38%
EKT	Euskaltel SA	3,97%	0,185%	0,012104	0,03%
TEF	Telefonica SA	3,51%	0,234%	0,009743	16,88%
ALM	Almirall SA	3,09%	0,039%	0,012409	0,04%
SONC	Sonae Capital SGPS SA	2,95%	0,166%	0,014412	0,04%
MCM	Miquel y Costas y Miquel SA	2,92%	0,080%	0,016414	0,94%
GCO	Grupo Catalana Occidente SA	2,82%	0,128%	0,010173	0,05%
FCC	Fomento de Construcciones y Contratas SA	2,78%	0,182%	0,013398	0,33%
IBS	Ibersol SGPS SA	2,43%	0,289%	0,021396	3,89%
ACX	Acerinox SA	2,24%	0,086%	0,016492	0,06%
JMT	Jeronimo Martins SGPS SA	2,11%	0,057%	0,011573	0,28%
MAS	Masmovil Ibercom SA	2,07%	0,708%	0,014907	24,95%
GAS	Gas Natural SDG SA	1,81%	0,203%	0,008594	9,22%
VIS	Viscofan SA	1,53%	-0,018%	0,009045	0,07%
SEM	Semapa SA	0,61%	0,074%	0,011391	0,05%
OLE	Deoleo SA	0,35%	-0,296%	0,037755	0,05%
CMO	Cementos Molins SA	0,30%	0,130%	0,014507	6,83%
TOTAL		92,96%			0,929623

Table A.4. azValor Iberia FI Optimal Risk Portfolio Results, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
GALP	Galp Energia SGPS SA	8,37%	-0,027%	0,009413	0,88%
ENO	Elecnor SA	7,94%	0,066%	0,008977	11,84%
TUB	Tubacex SA	7,77%	0,058%	0,015463	6,17%
TRE	Tecnicas Reunidas SA	6,33%	-0,043%	0,010924	5,37%
EGL	Mota-Engil	4,79%	0,239%	0,018889	2,91%
CCE	Coca-Cola European Partners PLC	4,19%	0,036%	0,006033	28,03%
ZOT	Zardoya Otis SA	4,02%	0,088%	0,012435	1,64%
MEL	Melia Hotels International SA	4,02%	0,185%	0,01226	1,72%
TEF	Telefonica SA	3,51%	0,234%	0,009743	5,19%
ALM	Almirall SA	3,09%	0,039%	0,012409	1,38%

MCM	Miquel y Costas y Miquel SA	2,92%	0,080%	0,016414	0,90%
GCO	Grupo Catalana Occidente SA	2,82%	0,128%	0,010173	3,44%
IBS	Ibersol SGPS SA	2,43%	0,289%	0,021396	2,31%
JMT	Jeronimo Martins SGPS SA	2,11%	0,057%	0,011573	5,61%
MAS	Masmovil Ibercom SA	2,07%	0,708%	0,014907	1,63%
VIS	Viscofan SA	1,53%	-0,018%	0,009045	6,50%
SEM	Semapa SA	0,61%	0,074%	0,011391	3,45%
OLE	Deoleo SA	0,35%	-0,296%	0,037755	0,97%
CMO	Cementos Molins SA	0,30%	0,130%	0,014507	3,03%
TOTAL		92,96%			0,929622

Table A.5. Summary quarterly results of azValor Iberia FI

Period of the year 2017	Return of the fund	Optimal return (return maximization)	Variance of the original portfolio	Variance of the optimal portfolio (risk minimization)
1st Quarter	0.079%	0.266%	0.00168%	0.00044%
2nd Quarter	0.127%	0.419%	0.00360%	0.00082%
3rd Quarter	-0.012%	0.135%	0.00218%	0.00088%
4th Quarter	0.072%	0.296%	0.00212%	0.00078%

Table A.6. Symphonia Azionario Small Cap Italia I Portfolio Composition, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.
IP	Interpump Group SpA	7,17%	0,003%	0,013802
BION	BB Biotech AG Ord	6,18%	-0,007%	0,011151
AMP	Amplifon SpA	5,25%	0,010%	0,015702
IMA	Industria Macchine Automatiche SpA	4,48%	-0,260%	0,013347
EIT	EI Towers SpA	3,45%	0,112%	0,011697
STS	Ansaldo Sts SpA	3,32%	0,127%	0,008276
REY	Reply SPA	3,28%	-0,130%	0,021608
MARR	Marr SpA	3,02%	-0,049%	0,012777
IF	Banca Ifis	2,90%	-0,174%	0,023238
BSS	Biesse SpA	2,39%	0,203%	0,01489
TIP	Tamburi Investment Partners SpA	1,96%	-0,089%	0,014677
DAL	Datalogic SPA	1,88%	0,071%	0,017981
AVIO	Avio SpA	1,70%	-0,009%	0,008926
ENAV	ENAV SpA	1,56%	0,246%	0,010532
FILA	FILA-Fabbrica Italiana Lapis ed Affini SpA	1,55%	0,078%	0,014398
OVS	OVS SpA	1,43%	-0,233%	0,011178
ELN	El.En. SpA	1,31%	0,173%	0,02379
SAL	Salini Impregilo SPA	1,21%	0,000%	0,017205
AGL	Autogrill SpA	1,19%	0,078%	0,012426
ZV	Zignago Vetro SpA	1,13%	0,015%	0,015035
BRE	Brembo SpA	1,12%	-0,187%	0,011687
IGD	Igd SIIQ SPA	1,09%	0,160%	0,019917
MT	Maire Tecnimont SpA	1,09%	-0,096%	0,02558
SO	Sogefi SpA	1,06%	-0,358%	0,027489
INDB	Indel B SpA	1,04%	0,302%	0,016382
LIT	Retelit SpA	0,94%	0,248%	0,017377
SAB	Sabaf	0,94%	-0,057%	0,017879
CEM	Cementir Holding SpA	0,90%	0,097%	0,011491
FKR	Falck Renewables SpA	0,89%	0,514%	0,025867
LUX	Luxottica Group SpA	0,87%	0,132%	0,012143
CERV	Cerved Information Solutions SpA	0,87%	0,099%	0,011442
GIMA	Gima TT SpA Ordinary Shares	0,84%	0,146%	0,016593
DLG	De'Longhi SPA	0,82%	-0,096%	0,022137
OJM	Openjobmetis SpA Agenzia per il Lavoro	0,81%	-0,035%	0,020954
SES	Sesa SpA	0,81%	-0,064%	0,015374
PST	Poste Italiane SpA	0,79%	0,014%	0,007421
ASC	Ascopiave SpA	0,76%	0,025%	0,012116
ANIM	Anima Holding SpA	0,75%	-0,196%	0,019995
SG	SAES Getters SPA	0,72%	-0,168%	0,01692
MONC	Moncler SpA	0,70%	0,113%	0,013337

CAI	Cairo Communication SpA	0,70%	-0,216%	0,014729
CARR	Carraro SPA	0,68%	-0,176%	0,029353
PAN	Panariagroup Industrie Ceramiche SpA	0,67%	-0,163%	0,019226
MN	Arnoldo Mondadori Editore SpA	0,65%	0,085%	0,027455
MZB	Massimo Zanetti Beverage Group SpA	0,65%	-0,246%	0,014864
IRE	Iren SpA	0,63%	0,162%	0,0136
LD	La Doria SpA	0,62%	0,294%	0,026716
US	UnipolSai SPA	0,61%	-0,019%	0,009152
REC	Recordati	0,61%	-0,075%	0,010566
ADB	Aeroporto Guglielmo Marconi di Bologna SpA	0,60%	0,141%	0,011751
BNS	Beni Stabili SpA SIIQ	0,58%	0,088%	0,014249
GEDI	GEDI Gruppo Editoriale SpA	0,57%	-0,087%	0,019559
RWAY	Rai Way SpA	0,54%	0,186%	0,019907
ELC	Elica SpA	0,53%	-0,018%	0,03286
PRI	Prima Industrie SPA	0,53%	-0,251%	0,025443
IWB	Italian Wine Brands SpA	0,52%	0,080%	0,009756
ECNL	Aquafil SpA	0,51%	0,125%	0,010684
SGR	SAES Getters SPA Rsp	0,50%	-0,168%	0,01692
LDO	Leonardo SpA	0,50%	-0,689%	0,030658
VAS	Vittoria Assicurazioni	0,49%	0,064%	0,011666
FBK	FinecoBank SpA	0,48%	0,215%	0,013681
BZUR	Buzzi Unicem SpA Rsp	0,48%	-0,013%	0,014695
MOL	Gruppo Mutuionline SpA	0,46%	0,010%	0,015035
EM	Emak SpA	0,45%	-0,539%	0,022672
CMB	Cembre SpA	0,45%	-0,221%	0,014591
LUVE	LU-VE SpA	0,42%	0,019%	0,018063
TES	Tesmec SpA	0,41%	0,072%	0,019104
RM	Reno de Medici SPA	0,35%	-0,141%	0,023331
TECN	Tecnoinvestimenti SpA	0,34%	0,157%	0,014963
815134	Wiit SpA	0,33%	0,045%	0,009496
IG	Italgas SpA	0,32%	0,118%	0,013284
BST	Banca Sistema SpA	0,31%	-0,057%	0,018176
GE	Gefran SpA	0,31%	-0,347%	0,037374
ON	Bio-on SpA	0,31%	0,102%	0,018749
BEC	B&C Speakers SpA	0,24%	-0,217%	0,025575
FNL	Finlogic SpA	0,22%	0,103%	0,021578
ISG	Isagro SpA	0,22%	-0,103%	0,037365
DIG	Digital360 SpA	0,10%	-0,00072	0,02292
TOTAL		92,03%		

Table A.7. Results of Symphonia Azionario Small Cap Italia I, 4th quarter of 2017

Average daily return of the portfolio	-0.007%
Variance of the portfolio	0.0000346

Table A.8. Symphonia Azionario Small Cap Italia I Optimal Return Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
EIT	EI Towers SpA	3,45%	0,112%	0,011697	2,50%
STS	Ansaldo Sts SpA	3,32%	0,127%	0,008276	5,38%
BSS	Biesse SpA	2,39%	0,203%	0,01489	6,16%
ENAV	ENAV SpA	1,56%	0,246%	0,010532	18,50%
ELN	El.En. SpA	1,31%	0,173%	0,02379	2,02%
IGD	Igd SIIQ SPA	1,09%	0,160%	0,019917	0,71%
INDB	Indel B SpA	1,04%	0,302%	0,016382	17,97%
LIT	Retelit SpA	0,94%	0,248%	0,017377	3,52%
FKR	Falck Renewables SpA	0,89%	0,514%	0,025867	15,92%
LUX	Luxottica Group SpA	0,87%	0,132%	0,012143	0,70%
ADB	Aeroporto Guglielmo Marconi di Bologna SpA	0,60%	0,141%	0,011751	2,15%
RWAY	Rai Way SpA	0,54%	0,186%	0,019907	7,98%
ECNL	Aquafil SpA	0,51%	0,125%	0,010684	3,93%
FBK	FinecoBank SpA	0,48%	0,215%	0,013681	4,58%
TOTAL		92,03%			92,03%

Table A.9. Symphonia Azionario Small Cap Italia I Optimal Risk Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
BION	BB Biotech AG Ord	6,18%	-0,007%	0,011151025	2,53%
IMA	Industria Macchine Automatiche SpA	4,48%	-0,260%	0,01334737	1,83%
EIT	EI Towers SpA	3,45%	0,112%	0,011697446	1,13%
STS	Ansaldo Sts SpA	3,32%	0,127%	0,008275549	17,23%
REY	Reply SPA	3,28%	-0,130%	0,02160754	0,58%
AVIO	Avio SpA	1,70%	-0,009%	0,008925892	3,96%
ENAV	ENAV SpA	1,56%	0,246%	0,010532102	1,23%
FILA	FILA-Fabbrica Italiana Lapis ed Affini SpA	1,55%	0,078%	0,014397975	0,51%
OVS	OVS SpA	1,43%	-0,233%	0,011178064	6,01%

SAL	Salini Impregilo SPA	1,21%	0,000%	0,01720543	2,81%
IGD	Igd SIIQ SPA	1,09%	0,160%	0,01991668	1,86%
SAB	Sabaf	0,94%	-0,057%	0,017879095	0,57%
SES	Sesa SpA	0,81%	-0,064%	0,015373617	0,89%
PST	Poste Italiane SpA	0,79%	0,014%	0,007421399	6,20%
ASC	Ascopiave SpA	0,76%	0,025%	0,012115892	1,03%
CAI	Cairo Communication SpA	0,70%	-0,216%	0,014729412	2,71%
MZB	Massimo Zanetti Beverage Group SpA	0,65%	-0,246%	0,014864185	8,60%
US	UnipolSai SPA	0,61%	-0,019%	0,009152442	10,49%
IWB	Italian Wine Brands SpA	0,52%	0,080%	0,00975559	2,77%
ECNL	Aquafil SpA	0,51%	0,125%	0,010684301	1,65%
LDO	Leonardo SpA	0,50%	-0,689%	0,030658354	0,07%
VAS	Vittoria Assicurazioni	0,49%	0,064%	0,01166557	5,04%
MOL	Gruppo Mutuionline SpA	0,46%	0,010%	0,015035242	1,76%
TES	Tesmec SpA	0,41%	0,072%	0,019103826	0,63%
815134	Wiit SpA	0,33%	0,045%	0,009496143	5,20%
IG	Italgas SpA	0,32%	0,118%	0,013284299	3,57%
GE	Gefran SpA	0,31%	-0,347%	0,037373599	0,80%
FNL	Finlogic SpA	0,22%	0,103%	0,021578181	0,38%
TOTAL		92,03%			92,03%

Table A.10. Summary quarterly results of Symphonia Azionario Small Cap Italia I

Period of the year 2017	Return of the fund	Optimal return (return maximization)	Variance of the original portfolio	Variance of the optimal portfolio (risk minimization)
1st Quarter	0.307%	0.523%	0.00366%	0.00139%
2nd Quarter	0.073%	0.472%	0.00557%	0.00102%
3rd Quarter	0.186%	0.413%	0.00297%	0.00082%
4th Quarter	-0.007%	0.251%	0.00346%	0.00107%

Table A.11. ID France Smidcaps I Portfolio Composition, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev
ESI	Esi Group SA	2,52%	0,270%	0,024529
ALFPC	Fountaine Pajot SA	2,50%	0,317%	0,015708
OPN	Groupe Open SA	2,44%	0,210%	0,012377
TRI	Trigano SA	2,44%	0,200%	0,016387
PHA	Pharmagest Interactive	2,42%	-0,181%	0,016521
DIREN	Direct Energie	2,40%	-0,274%	0,024657
SFPI	Groupe SFPI SA	2,38%	-0,019%	0,017524
TES	Tessi	2,37%	0,009%	0,009808
INF	Infotel SA	2,36%	0,223%	0,013382
ALFOC	Focus Home Interactive SA	2,33%	0,259%	0,01717
SII	Societe Pour L'Informatique Industrielle	2,33%	0,054%	0,011295
MAGIS	Ymagis	2,33%	-0,181%	0,009787
ILD	Iliad SA	2,31%	-0,182%	0,010793
AKA	Akka Technologies	2,30%	-0,094%	0,01237
KOF	Kaufman & Broad SA	2,29%	-0,041%	0,010616
PREC	Precia SA	2,29%	0,078%	0,012119
ALDEL	Delfingen Industry SA	2,28%	0,063%	0,014146
MFC	Maisons France Confort	2,28%	0,042%	0,012876
ALGIL	Groupe Guillin SA	2,28%	-0,342%	0,025824
EXE	Exel Industries SA	2,27%	0,155%	0,012785
JCQ	Jacquet Metal Service	2,27%	-0,021%	0,016509
DVT	Devoteam SA	2,27%	-0,063%	0,017778
FNAC	Fnac Darty SA	2,22%	0,344%	0,011519
TFF	Tonnellerie François Frères	2,21%	0,225%	0,015646
DLTA	Delta Plus Group	2,21%	0,112%	0,017646
AUB	Aubay	2,19%	0,024%	0,014671
RIN	Vilmorin & Cie	2,19%	0,256%	0,018784
SDG	Synergie SE	2,18%	-0,021%	0,012984
ORP	Orpea SA	2,17%	-0,026%	0,009382
BIG	Bigben Interactive	2,15%	0,527%	0,025283
FLE	Fleury Michon	2,15%	-0,004%	0,028122
PERR	Gerard Perrier Industrie SA	2,14%	0,096%	0,009336
SBT	Oeneo	2,09%	0,112%	0,013181
PVL	Plastiques du Val-de-Loire	2,06%	-0,191%	0,022157
ASP	AST Groupe	2,02%	0,034%	0,016478
MMT	Metropole Television SA	2,02%	0,161%	0,013456
IPN	Ipsen SA	1,92%	-0,178%	0,017234
LOUP	L D C SA	1,90%	0,061%	0,015232
MRN	Mersen SA	1,49%	0,348%	0,022256
MND	Montagne et Neige Developpement SACA	1,38%	0,257%	0,021501
HCO	High Co	1,34%	0,036%	0,018311
IGE	IGE Plus XAO SA	1,28%	0,375%	0,019833

LSS	Lectra	1,26%	-0,073%	0,016879
ATI	Actia group	1,15%	-0,080%	0,017767
MTU	Manitou BF SA	1,14%	-0,046%	0,018839
AURS	Aures Technologies SA	1,03%	-0,058%	0,022255
SAMS	Samse SA	0,94%	0,153%	0,003303
ALUMT	Umanis NR	0,39%	0,303%	0,021444
NRO	Neurones	0,27%	0,047%	0,010188
TOTAL		97,17%		

Table A.12. Results of ID France Smidcaps I, 4th quarter of 2017

Average daily return of the portfolio	0.060%
Variance of the portfolio	0.0000184

Table A.13. ID France Smidcaps I Optimal Return Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev	Optimal weight
ESI	Esi Group SA	2,52%	0,270%	0,024529	0,28%
ALFPC	Fountaine Pajot SA	2,50%	0,317%	0,015708	4,18%
OPN	Groupe Open SA	2,44%	0,210%	0,012377	12,53%
ALFOC	Focus Home Interactive SA	2,33%	0,259%	0,01717	6,05%
ALDEL	Delfingen Industry SA	2,28%	0,063%	0,014146	0,06%
EXE	Exel Industries SA	2,27%	0,155%	0,012785	0,12%
JCQ	Jacquet Metal Service	2,27%	-0,021%	0,016509	0,02%
DVT	Devoteam SA	2,27%	-0,063%	0,017778	0,02%
FNAC	Fnac Darty SA	2,22%	0,344%	0,011519	14,04%
TFF	Tonnellerie François Frères	2,21%	0,225%	0,015646	0,17%
DLTA	Delta Plus Group	2,21%	0,112%	0,017646	0,57%
RIN	Vilmorin & Cie	2,19%	0,256%	0,018784	4,36%
BIG	Bigben Interactive	2,15%	0,527%	0,025283	3,29%
FLE	Fleury Michon	2,15%	-0,004%	0,028122	0,02%
PERR	Gerard Perrier Industrie SA	2,14%	0,096%	0,009336	6,43%
MRN	Mersen SA	1,49%	0,348%	0,022256	10,30%
HCO	High Co	1,34%	0,036%	0,018311	0,47%
IGE	IGE Plus XAO SA	1,28%	0,375%	0,019833	15,03%
SAMS	Samse SA	0,94%	0,153%	0,003303	13,56%
ALUMT	Umanis NR	0,39%	0,303%	0,021444	5,67%
TOTAL		97,17%			97,17%

Table A.14. ID France Smidcaps I Optimal Risk Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev	Optimal weight
PHA	Pharmagest Interactive	2,42%	-0,181%	0,016521	1,17%
DIREN	Direct Energie	2,40%	-0,274%	0,024657	0,19%
TES	Tessi	2,37%	0,009%	0,009808	6,83%
ALFOC	Focus Home Interactive SA	2,33%	0,259%	0,01717	0,61%
SII	Societe Pour L'Informatique Industrielle	2,33%	0,054%	0,011295	2,25%
MAGIS	Ymagis	2,33%	-0,181%	0,009787	5,30%
ILD	Iliad SA	2,31%	-0,182%	0,010793	8,77%
AKA	Akka Technologies	2,30%	-0,094%	0,01237	0,26%
PREC	Precia SA	2,29%	0,078%	0,012119	1,70%
ALDEL	Delfingen Industry SA	2,28%	0,063%	0,014146	2,26%
MFC	Maisons France Confort	2,28%	0,042%	0,012876	3,14%
ALGIL	Groupe Guillin SA	2,28%	-0,342%	0,025824	0,57%
EXE	Exel Industries SA	2,27%	0,155%	0,012785	1,05%
TFF	Tonnellerie François Frères	2,21%	0,225%	0,015646	1,82%
DLTA	Delta Plus Group	2,21%	0,112%	0,017646	2,95%
ORP	Orpea SA	2,17%	-0,026%	0,009382	4,06%
FLE	Fleury Michon	2,15%	-0,004%	0,028122	1,43%
PERR	Gerard Perrier Industrie SA	2,14%	0,096%	0,009336	6,24%
SBT	Oeneo	2,09%	0,112%	0,013181	0,58%
HCO	High Co	1,34%	0,036%	0,018311	2,68%
IGE	IGE Plus XAO SA	1,28%	0,375%	0,019833	2,61%
ATI	Actia group	1,15%	-0,080%	0,017767	2,32%
AURS	Aures Technologies SA	1,03%	-0,058%	0,022255	1,13%
SAMS	Samse SA	0,94%	0,153%	0,003303	35,13%
ALUMT	Umanis NR	0,39%	0,303%	0,021444	2,11%
TOTAL		97,17%			97,17%

Table A.15. Summary quarterly results of ID France Smidcaps I

Period of the year 2017	Return of the fund	Optimal return (return maximization)	Variance of the original portfolio	Variance of the optimal portfolio (risk minimization)
1st Quarter	0.145%	0.277%	0.00111%	0.00020%
2nd Quarter	0.179%	0.431%	0.00465%	0.00126%
3rd Quarter	0.059%	0.102%	0.00108%	0.00011%
4th Quarter	0.060%	0.271%	0.00184%	0.00036%

Table A.16. BERENBERG-1590-Aktien Mittelstand I Portfolio Composition, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.
WDI	Wirecard AG	3,50%	0,305%	0,01709
STM	Stabilus SA	3,11%	-0,016%	0,019223
P1Z	PATRIZIA Immobilien AG	3,08%	0,148%	0,017099
ZAL	Zalando SE	3,07%	0,080%	0,015751
GLJ	Grenkeleasing AG	3,04%	0,078%	0,014185
O1BC	XING SE	2,99%	0,018%	0,015578
JUN3	Jungheinrich AG	2,96%	0,039%	0,016158
DUE	Duerr AG	2,93%	-0,086%	0,014881
NOEJ	NORMA Group SE	2,93%	0,013%	0,016809
UTDI	United Internet AG	2,92%	0,138%	0,013517
SY1	Symrise AG	2,84%	0,176%	0,010401
VACN	VAT Group AG	2,79%	0,128%	0,013489
SAX	Stroeer SE & Co KGaA	2,70%	0,193%	0,016362
RAA	Rational AG	2,51%	-0,115%	0,012472
DRI	Drillisch AG	2,42%	0,243%	0,012056
MOR	MorphoSys AG	2,40%	0,161%	0,020252
FIE	Fielmann AG	2,38%	0,006%	0,010196
NEM	Nemetschek SE	2,37%	0,151%	0,019556
EVD	CTS Eventim AG & Co. KGaA	2,35%	0,065%	0,013041
AFX	Carl Zeiss Meditec AG	2,35%	0,269%	0,015041
HYQ	Hypoport AG	2,33%	0,012%	0,028126
GYC	Grand City Properties SA	2,12%	0,155%	0,01074
SRT3	Sartorius AG Pfd Shs - Non-voting	2,11%	-0,027%	0,020323
N7G	Nanogate SE	2,06%	-0,163%	0,017246
FPE3	Fuchs Petrolub SE Pfd Shs - Non-voting	1,97%	-0,108%	0,01009
VQT	va-Q-tec AG	1,95%	-0,273%	0,030382
LNSX	Sixt Leasing SE	1,77%	-0,194%	0,025184
RIB	RIB Software SE	1,73%	0,546%	0,02573
VOS	Vossloh AG	1,73%	-0,321%	0,015314
VAO	Vapiano SE	1,69%	0,102%	0,01988
RKET	Rocket Internet SE	1,66%	-0,019%	0,019423
ADN1	Adesso AG	1,66%	0,128%	0,017286
SHF	SNP Schneider-Neureither & Partner SE	1,65%	-0,057%	0,023458
COK	CANCOM SE	1,55%	0,142%	0,014396
BNN	Brain Biotechnology Research and Information Network AG	1,49%	0,223%	0,025187
QGEN	Qiagen NV	1,46%	-0,013%	0,02064
BC8	Bechtle AG	1,36%	0,172%	0,018412
SKB	Koenig & Bauer AG	1,31%	-0,171%	0,010472
KSC	Kps AG	1,17%	-0,269%	0,030141
EVT	Evotec AG	1,13%	-0,505%	0,050078
HFG	HelloFresh SE Bearer Shares	0,95%	0,346%	0,034204

AAG	Aumann AG	0,72%	-0,400%	0,030636
ISR	Isra Vision AG	0,57%	0,529%	0,029128
WIG1	SPORTTOTAL AG	0,45%	0,051%	0,024524
TOTAL		92,26%		

Table A.17. Results of BERENBERG-1590-Aktien Mittelstand I, 4th quarter of 2017

Average daily return of the portfolio	0.051%
Variance of the portfolio	0.0000382

Table A.18. BERENBERG-1590-Aktien Mittelstand I Optimal Return Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
GLJ	Grenkeleasing AG	3,04%	0,078%	0,014185	10,57%
SY1	Symrise AG	2,84%	0,176%	0,010401	10,22%
SAX	Stroeer SE & Co KGaA	2,70%	0,193%	0,016362	5,12%
DRI	Drillisch AG	2,42%	0,243%	0,012056	10,45%
MOR	MorphoSys AG	2,40%	0,161%	0,020252	5,91%
AFX	Carl Zeiss Meditec AG	2,35%	0,269%	0,015041	12,56%
GYC	Grand City Properties SA	2,12%	0,155%	0,01074	17,85%
RIB	RIB Software SE	1,73%	0,546%	0,02573	10,78%
HFG	HelloFresh SE Bearer Shares	0,95%	0,346%	0,034204	5,66%
ISR	Isra Vision AG	0,57%	0,529%	0,029128	3,16%
TOTAL		92,26%			92,26%

Table A.19. BERENBERG-1590-Aktien Mittelstand I Optimal Risk Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
P1Z	PATRIZIA Immobilien AG	3,08%	0,148%	0,017099	5,30%
GLJ	Grenkeleasing AG	3,04%	0,078%	0,014185	15,98%
SY1	Symrise AG	2,84%	0,176%	0,010401	7,09%
SAX	Stroeer SE & Co KGaA	2,70%	0,193%	0,016362	0,32%
DRI	Drillisch AG	2,42%	0,243%	0,012056	5,14%
MOR	MorphoSys AG	2,40%	0,161%	0,020252	7,10%
EVD	CTS Eventim AG & Co. KGaA	2,35%	0,065%	0,013041	0,41%
AFX	Carl Zeiss Meditec AG	2,35%	0,269%	0,015041	4,59%

GYC	Grand City Properties SA	2,12%	0,155%	0,01074	13,44%
SRT3	Sartorius AG Pfd Shs	2,11%	-0,027%	0,020323	3,03%
FPE3	Fuchs Petrolub SE Pfd Shs	1,97%	-0,108%	0,01009	7,41%
VOS	Vossloh AG	1,73%	-0,321%	0,015314	10,22%
SKB	Koenig & Bauer AG	1,31%	-0,171%	0,010472	5,01%
KSC	Kps AG	1,17%	-0,269%	0,030141	2,41%
HFG	HelloFresh SE Bearer Shares	0,95%	0,346%	0,034204	4,81%
TOTAL		92,26%			92,26%

Table A.20. Summary quarterly results of BERENBERG-1590-Aktien Mittelstand I

Period of the year 2017	Return of the fund	Optimal return (return maximization)	Variance of the original portfolio	Variance of the optimal portfolio (risk minimization)
1st Quarter	0.174%	0.316%	0.00300%	0.00107%
2nd Quarter	0.123%	0.332%	0.00585%	0.00308%
3rd Quarter	0.164%	0.338%	0.00342%	0.00120%
4th Quarter	0.051%	0.228%	0.00382%	0.00144%

Table A.21. Old Mutual UK Smlr Coms Foc U1 GBP Inc Portfolio Composition, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.
PRSM	Blue Prism Group PLC	6,33%	0,446%	0,04307
FEVR	Fevertree Drinks PLC	5,37%	0,106%	0,029548
MCGN	Microgen PLC	4,00%	-0,035%	0,025375
AFX	Alpha FX Group PLC	3,81%	0,050%	0,018796
ASCL	Ascential PLC	3,69%	0,190%	0,012687
BOO	boohoo.com PLC	3,50%	-0,153%	0,024309
CCFS	Charter Court Financial Services Group PLC	3,11%	0,269%	0,015255
PURP	Purplebricks Group PLC	3,09%	0,172%	0,034342
CLIN	Clinigen Group PLC	3,07%	-0,038%	0,018513
CVR	Conviviality PLC	3,05%	-0,024%	0,013218
JSG	Johnson Service Group PLC	2,81%	-0,029%	0,013754
OSB	Onesavings Bank PLC	2,69%	0,049%	0,015734
MSLH	Marshalls PLC	2,58%	0,075%	0,012201
MGP	Medica Group PLC	2,57%	0,092%	0,022443
BUR	Burford Capital Ltd	2,47%	0,188%	0,019147
VCP	Victoria PLC	2,46%	0,454%	0,01665
TENG	Ten Lifestyle Group PLC	1,97%	0,203%	0,017473
GYG	GYG PLC	1,93%	-0,004%	0,019552
G4M	Gear4music (Holdings) PLC	1,84%	-0,074%	0,023948
WKP	Workspace Group PLC	1,82%	0,204%	0,015358
SNN	Sanne Group PLC	1,74%	0,040%	0,013529
EAH	Eco Animal Health Group PLC	1,69%	-0,078%	0,01093
TCAP	TP ICap PLC	1,64%	0,032%	0,012714
KWS	Keywords Studios PLC	1,60%	0,263%	0,026578
XPP	XP Power Ltd	1,54%	0,347%	0,014686
BOY	Bodycote PLC	1,45%	0,001%	0,012283
CRST	Crest Nicholson Holdings PLC	1,43%	-0,005%	0,019207
VSVS	Vesuvius PLC	1,43%	-0,009%	0,012366
FOOT	Footasylum PLC Ordinary Shares	1,35%	1,152%	0,038311
RWA	Robert Walters PLC	1,32%	0,115%	0,023767
APGN	Applegreen PLC	1,31%	-0,171%	0,009807
SYNT	Synthomer PLC	1,27%	0,015%	0,009716
FDM	FDM Group (Holdings) PLC	1,19%	-0,035%	0,016072
ECM	Electrocomponents PLC	1,16%	0,022%	0,012634
JOUL	Joules Group PLC	1,03%	0,061%	0,017176
SBRE	Sabre Insurance Group PLC	0,96%	1,112%	0,035926
SWL	Swallowfield PLC	0,83%	-0,007%	0,023576
AFHP	AFH Financial Group PLC	0,83%	0,177%	0,014996
GLEN	Glencore PLC	0,80%	0,220%	0,014979
HOTC	Hotel Chocolat Group PLC	0,78%	0,324%	0,022625
RIO	Rio Tinto PLC	0,77%	0,211%	0,014124

DATA	GlobalData PLC	0,70%	0,031%	0,008268
ZOO	Zoo Digital Group PLC	0,63%	0,885%	0,060295
FFI	FFI Holdings PLC	0,58%	0,086%	0,010301
GHT	Gresham Technologies PLC	0,56%	-0,024%	0,016112
HFG	Hilton Food Group PLC	0,54%	0,289%	0,015877
MAB1	Mortgage Advice Bureau (Holdings) PLC	0,50%	0,146%	0,015599
FFX	FairFX Group PLC	0,49%	0,165%	0,019608
SRT	SRT Marine Systems PLC	0,48%	-0,069%	0,019046
BOKU	Boku Inc Ordinary Shares	0,42%	0,888%	0,053864
KMK	Kromek Group PLC	0,39%	0,159%	0,034585
ANTO	Antofagasta PLC	0,28%	0,102%	0,015131
OPM	1pm PLC	0,01%	-0,165%	0,018143
WAND	Wandisco PLC	0,01%	-0,516%	0,021718
TOTAL		93,90%		

Table A.22. Results of Old Mutual UK Smlr Coms Foc U1 GBP Inc, 4th quarter of 2017

Average daily return of the portfolio	0.131%
Variance of the portfolio	0.0000418

Table A.23. Old Mutual UK Smlr Coms Foc U1 GBP Inc Q4 Optimal Profit Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
PRSM	Blue Prism Group PLC	6,33%	0,446%	0,04307	0,005%
FEVR	Fevertree Drinks PLC	5,37%	0,106%	0,029548	5,475%
AFX	Alpha FX Group PLC	3,81%	0,050%	0,018796	8,471%
CCFS	Charter Court Financial Services Group PLC	3,11%	0,269%	0,015255	2,812%
MGP	Medica Group PLC	2,57%	0,092%	0,022443	10,932%
VCP	Victoria PLC	2,46%	0,454%	0,01665	9,662%
WKP	Workspace Group PLC	1,82%	0,204%	0,015358	4,702%
FOOT	Footasylum PLC Ordinary Shares	1,35%	1,152%	0,038311	15,340%
SBRE	Sabre Insurance Group PLC	0,96%	1,112%	0,035926	24,401%
SWL	Swallowfield PLC	0,83%	-0,007%	0,023576	1,145%
AFHP	AFH Financial Group PLC	0,83%	0,177%	0,014996	0,687%
FFX	FairFX Group PLC	0,49%	0,165%	0,019608	1,189%
BOKU	Boku Inc Ordinary Shares	0,42%	0,888%	0,053864	0,147%
KMK	Kromek Group PLC	0,39%	0,159%	0,034585	8,925%
TOTAL		93,90%			93,896%

Table A.24. Old Mutual UK Smlr Coms Foc U1 GBP Inc Optimal Risk Portfolio, 4th quarter of 2017

Ticker	Name	Weight	E (Return)	Std dev.	Optimal weight
FEVR	Fevertree Drinks PLC	5,37%	0,106%	0,029548	0,50%
AFX	Alpha FX Group PLC	3,81%	0,050%	0,018796	8,46%
BOO	boohoo.com PLC	3,50%	-0,153%	0,024309	4,05%
MGP	Medica Group PLC	2,57%	0,092%	0,022443	5,05%
G4M	Gear4music (Holdings) PLC	1,84%	-0,074%	0,023948	0,91%
WKP	Workspace Group PLC	1,82%	0,204%	0,015358	1,21%
EAH	Eco Animal Health Group PLC	1,69%	-0,078%	0,01093	29,54%
KWS	Keywords Studios PLC	1,60%	0,263%	0,026578	1,93%
APGN	Applegreen PLC	1,31%	-0,171%	0,009807	15,34%
FDM	FDM Group (Holdings) PLC	1,19%	-0,035%	0,016072	2,23%
SBRE	Sabre Insurance Group PLC	0,96%	1,112%	0,035926	15,58%
SWL	Swallowfield PLC	0,83%	-0,007%	0,023576	3,27%
GHT	Gresham Technologies PLC	0,56%	-0,024%	0,016112	0,58%
KMK	Kromek Group PLC	0,39%	0,159%	0,034585	3,74%
WAND	Wandisco PLC	0,01%	-0,516%	0,021718	1,51%
TOTAL		93,90%			93,90%

Table A.25. Summary quarterly results of Old Mutual UK Smlr Coms Foc U1 GBP Inc

Period of the year 2017	Return of the fund	Optimal return (return maximization)	Variance of the original portfolio	Variance of the optimal portfolio (risk minimization)
1st Quarter	0.297%	0.502%	0.00275%	0.00070%
2nd Quarter	0.274%	0.540%	0.00651%	0.00151%
3rd Quarter	0.208%	0.418%	0.00286%	0.00085%
4th Quarter	0.131%	0.548%	0.00418%	0.00168%